Probability Investigation: The Law of Large Numbers

The idea that the proportion of the outcomes approaches the theoretical probability over a large number of trials is the Law of Large Numbers. Remember that for a coin toss it is the fraction of heads that is expected to approach 0.5. Thus 53 heads out of 100 trials is considered closer to the prediction than 6 out of 10, because 0.53 is closer to 0.5 than 0.6 is to 0.5, even though 6 is closer to 5 (difference = 1) than 53 is to 50 (difference = 3).

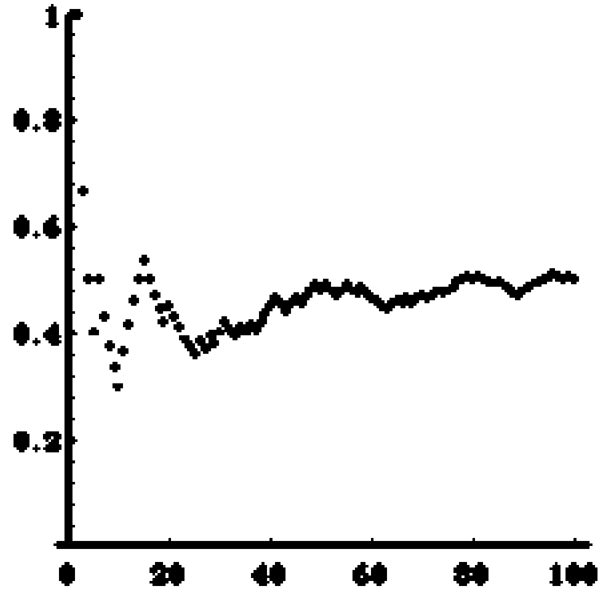
1. This is a graph of the fraction of heads in k coin tosses (computer simulated) for k = 1 - 100. For example, after 3 tosses the fraction was 0.67, so 2 heads and 1 tail had been obtained.

a) How many heads after 1 toss?

b) How many heads after 4 tosses?

c) How many heads after 88 tosses?

2. Discuss how this graph illustrates the Law of Large Numbers. In particular explain why your answers to 1(b) and 1(c) do not contradict the Law of Large Numbers.



You are going to use the Pascal Triangle to analyze why the Law of Large numbers is true.

Recall that row n lists the binomial coefficients $\binom{n}{k}$, which is the number of ways k heads can occur in n tosses of a coins. This coefficient can be computed on your TI-73 calculator using n nCp k, where nCp is found by choosing the math menu, probability menu, #4. A row of the Pascal Triangle can also be computed from the row above by adding the two terms above each new entry. The Pascal Triangle is also listed in your text on p. 21.

3. Write out the first 10 rows of the Pascal Triangle on another sheet of paper..

4. Explain why the probability of getting k heads in n tosses is $2^{-n}\binom{n}{k}$.

5. What is the probability of getting exactly 0.5n heads in n tosses for n=

a) 1 b) 2 c) 4 d) 6 e) 8 f) 10

6. Why do your answers to 4 not contradict the Law of Large Numbers?

In order to have the proportion of heads in n trials be between 0.4 and 0.6 the number of heads in n trails needs to be between 0.4n and 0.6n. For n=4 this means between 1.6 and 2.4, i.e., there must be 2 heads. There are 6 ways out of 16 that this can occur, so the probability of this is

$2^{-4}\binom{4}{2} = \frac{6}{16} = 0.375$. Record your answers to the next question on the back of this page.

7. For each n = 5, 6, 7, 8, 10, 20 find

a) the numbers of heads that result in the fraction of heads between 0.4 and 0.6 (inclusive).

b) The number of ways this can happen. c) The probability of this happening.