H1: For \( x \in \mathbb{R}^n \), \( 0 \cdot x = 0 = x \cdot 0 \).

H2: For \( x, y \in \mathbb{R}^n \), \((ax+by) \cdot (ax+by) = a^2 x \cdot x + 2ab x \cdot y + b^2 y \cdot y \).

H5: Let \( A \) be \( m \times k \) and \( B \) be \( k \times n \). Then the \( j \)th column of \( AB \) is \( A \) times the \( j \)th column of \( B \) and the \( i \)th row of \( AB \) is the \( i \)th row of \( A \) times \( B \).

H6: (cf. 1.13): For an \( n \times n \) matrix \( A \), let \( H = \frac{1}{2} \left( A + A^T \right) \) and \( V = \frac{1}{2} \left( A - A^T \right) \).

(1) \( A = H + V \)
(2) \( H \) is symmetric
(3) \( V \) is skew-symmetric
(4) If \( A = H' + V' \) with \( H' \) symmetric and \( V' \) skew-symmetric, then \( H' = H \) and \( V' = V \).

H7: Let \( A \) be a linear system and let \( [C|d] = \text{RREF}([A|b]) \).

(1) If there is a leading 1 in \( d \), then \( Ax = B \) is inconsistent.
(2) If there is not a leading 1 in \( d \) and there is a leading 1 in each column of \( C \) then \( Ax = B \) has a unique solution.
(3) If there is not a leading 1 in \( d \) and there is a column of \( C \) that does not contain a leading 1, then \( Ax = B \) has infinitely many solutions.

H7a: Let \( A \) be an \( m \times n \) matrix, \( b \) an \( n \times 1 \) vector and

1. If \( x_1, x_2 \) are solutions to \( Ax = b \), then \( x_1 - x_2 \) is a solution to \( Ax = 0 \).
2. If \( x_1 \) is a solution to \( Ax = b \) and \( x_0 \) is a solution to \( Ax = 0 \) then \( x_1 + cx_0 \) is a solution to \( Ax = b \) for any real number \( c \).

H7b: (cf. 2.3) The systems \( Ax = b \) and \( Cx = d \) are equivalent iff \( [A|b] \) and \( [C|d] \) are row equivalent.

H8: If \( C \) is obtained from \( A \) by one elementary row operation, then \( A \) can be obtained from \( C \) by one elementary row operation (reverse operation).

H9: \( \text{RREF}(A) = E_k \ldots E_1 A \) where \( E_1, \ldots, E_k \) are elementary matrices.

H9a: If \( A \) is \( m \times n \), rank \( A \leq \min\{m,n\} \)

H10: The inverse of an elementary matrix is an elementary matrix.

H10a: If \( A_1, \ldots, A_k \) are invertible matrices then \( (A_1, \ldots, A_k)^{-1} = A_k^{-1} \ldots A_1^{-1} \)

H11: Let \( A \) be an \( n \times n \) matrix. The following are equivalent:

(1) \( A \) is invertible.
(2) Exists \( n \times n \) \( C \) such that \( CA = I \)
(3) Exists \( n \times n \) \( D \) such that \( AD = I \)
(4) For all \( n \times 1 \) \( b \), \( Ax = b \) is consistent
(5) \( Ax = 0 \) has a unique solution
(6) For all \( n \times 1 \) \( b \), \( Ax = b \) has a unique solution
(7) Rank \( A = n \)
(8) \( \text{RREF}(A) = I \)
(9) \( A \) is a product of elementary matrices.

H12: If \( AB = I \) then \( A \) and \( B \) are invertible and \( B = A^{-1} \).