

Class: Nonnegative P_0 -Matrices

Status: Some progress.

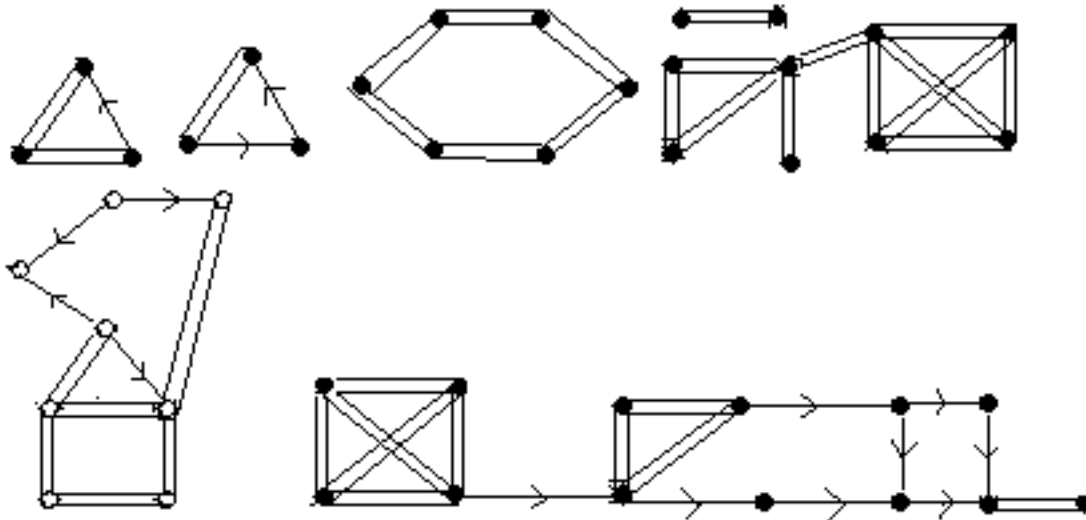
Definitions

- A matrix is a nonnegative P_0 -matrix if and only if every entry is nonnegative and every principal minor is nonnegative.
- The partial matrix B is a partial nonnegative P_0 -matrix if and only if every fully specified principal submatrix of B is a nonnegative P_0 -matrix and all specified entries are nonnegative.

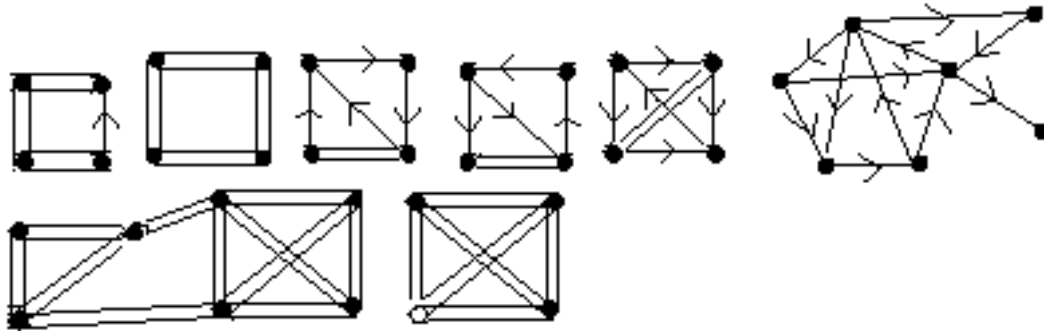
Results:

- A positionally symmetric pattern that includes all diagonal positions whose graph is block-clique has nonnegative P_0 -completion [FJTU].
- A pattern that omits all diagonal positions has nonnegative P_0 -completion. A positionally symmetric pattern has nonnegative P_0 -completion only if for every component H of its pattern-graph, either the pattern omits all diagonal positions corresponding to vertices in H , or includes all diagonal positions corresponding to vertices in H [H4].
- A pattern has nonnegative P_0 -completion if and only if every strongly connected and nonseparable induced submardigraph of its pattern mardigraph [H4].
- A positionally symmetric pattern that includes all diagonal positions and whose graph is an n -cycle does not have nonnegative P_0 -completion for $n = 4$ [H4] but does have nonnegative P_0 -completion for $n \geq 5$ [CDHKNS].
- All patterns for 2×2 and 3×3 matrices that include all diagonal positions have nonnegative P_0 -completion. A pattern for 4×4 matrices that includes all diagonal positions has nonnegative P_0 -completion if and only if it does not contain a 4-cycle or is the complete digraph on 4-vertices. [CDHKNS].

Examples: Have Nonnegative P_0 -completion



Examples: Do not have Nonnegative P_0 -completion



References:

- [CDHKNS] J.-Y. Choi, L. M. DeAlba, L. Hogben, B. Kivunge, S. Nordstrom, M. Shedenhelm, The Non-Negative P_0 -Matrix Completion Problem, *Electronic Journal of Linear Algebra* 10 (2003): 46-59.
- [FJTU] S. M. Fallat, C. R. Johnson, J. R. Torregrosa, and A. M. Urbano, P-matrix Completions under weak symmetry assumptions, *Linear Algebra and Its Applications* 312 (2000), 73-91.
- [H4] L. Hogben, Graph theoretic methods for matrix completion problems, *Linear Algebra and Its Applications* **328** (2001) 161-202, available electronically in PDF format at <http://www.math.iastate.edu/lhogben/research/GraphMatrixCompletion.pdf>