

Class: Nonnegative P-Matrices and Positive P-matrices

Status: Some progress- all known results are the same for positive P- and nonnegative P-matrices.

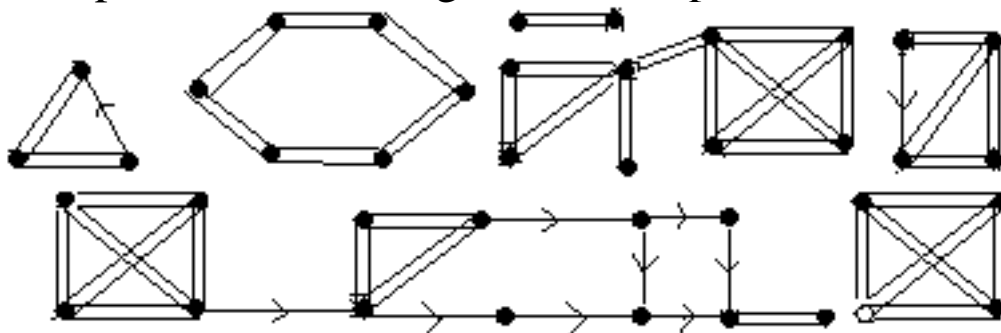
Definitions

- A matrix is a nonnegative (positive) P-matrix if and only if every entry is nonnegative (positive) and every principal minor is nonnegative.
- The partial matrix B is a partial nonnegative (positive) P-matrix if and only if every fully specified principal submatrix of B is a nonnegative (positive) P-matrix and all specified entries are nonnegative (positive).

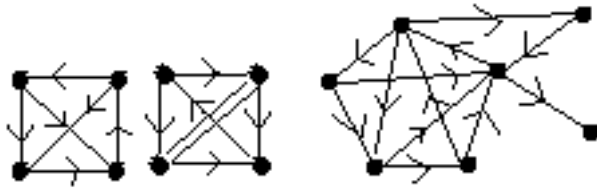
Results:

- A pattern has nonnegative (positive) P-completion if and only if the principal subpattern determined by the specified diagonal positions has nonnegative (positive) P-completion. The remaining results thus assume all diagonal positions are included in the pattern.
- A positionally symmetric pattern whose graph is block-clique has nonnegative (positive) P-completion [FJTU].
- A pattern has nonnegative (positive) P-completion if and only if every strongly connected and nonseparable induced subdigraph of its pattern digraph has nonnegative (positive) P-completion [H4].
- A positionally symmetric pattern that includes all diagonal positions and whose graph is an n-cycle has nonnegative (positive) P-completion [FJTU, H4].
- All patterns for 2×2 and 3×3 matrices have nonnegative (positive) P-completion [FJTU, H4].
- All patterns for 2×2 , 3×3 and 4×4 matrices have been classified as to nonnegative (positive) P-completion [HES], [JTU].
- A pattern whose digraph is minimally chordal symmetric Hamiltonian does not have nonnegative (positive) P-completion [HES].
- Any pattern that has nonnegative P_0 -completion has nonnegative P-completion [H6].
- Any pattern that has nonnegative P-completion has positive P-completion [H7].

Examples: Have Nonnegative P-completion



Examples: Do not have Nonnegative P-completion



References:

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- [HES] L. Hogben, J. Evers, and S. Shaner, The Positive and Nonnegative P-matrix Completion Problems, preprint available electronically in PDF format at <http://www.math.iastate.edu/lhogben/research/nnP.pdf>
- [JTU] C. Jordán, J. R. Torregrosa, and A. M. Urbano, Completions of partial P-matrices with acyclic or non-acyclic associated graph, *Linear Algebra Appl.* 368 (2003), 25–51.