

Class: Positive Semi-Definite Matrices (PSD-Matrices)

Note: Since this class is symmetric, all patterns are positionally symmetric and diagrams are graphs rather than digraphs.

Status: Done

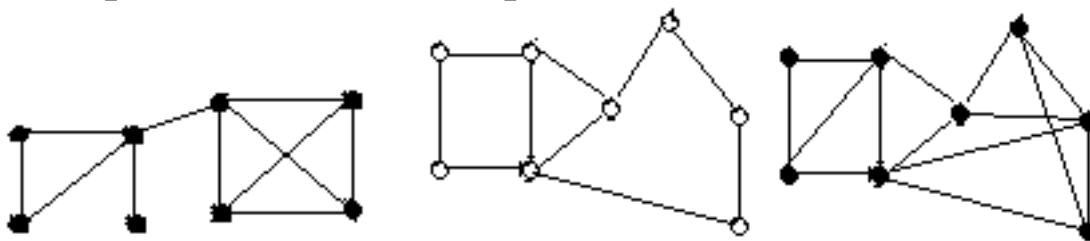
Definitions

- A matrix A is positive semi-definite if and only if A is symmetric and for all x , $x^T A x \geq 0$ if and only if A is symmetric and all principal minors are nonnegative.
- The partial matrix B is a partial positive semi-definite matrix if and only if every fully specified principal submatrix of B is a positive semi-definite-matrix, and whenever b_{ij} is specified then so is b_{ji} and $b_{ji} = b_{ij}$.

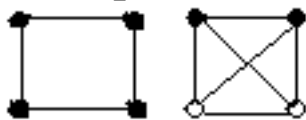
Results:

- A pattern that includes all diagonal positions has PSD-completion if and only if its pattern-graph is chordal [GJSW], [JS4].
- A pattern has PSD-completion if and only if the principal subpattern corresponding to each component H of its pattern-graph either omits all diagonal positions or includes all diagonal positions and H is chordal [H4].

Examples: Have PSD-completion



Examples: Do not have PSD-completion



References:

- [GJSW] R. Grone, C. R. Johnson, E. M. Sá, and H. Wolkowicz, Positive Definite Completions of Partial Hermitian Matrices, *Linear Algebra and Its Applications* 58 (1984), 109-124.
- [JS4] C. R. Johnson and R. L. Smith, The positive definite completion problem relative to a subspace, *Linear Algebra and Its Applications* 307 (2000), 1-14
- [H4] L. Hogben, Graph theoretic methods for matrix completion problems, *Linear Algebra and Its Applications* 328 (2001) 161-202, available electronically in PDF format at <http://www.math.iastate.edu/lhogben/research/GraphMatrixCompletion.pdf>