

Class: M-Matrices

Status: Done

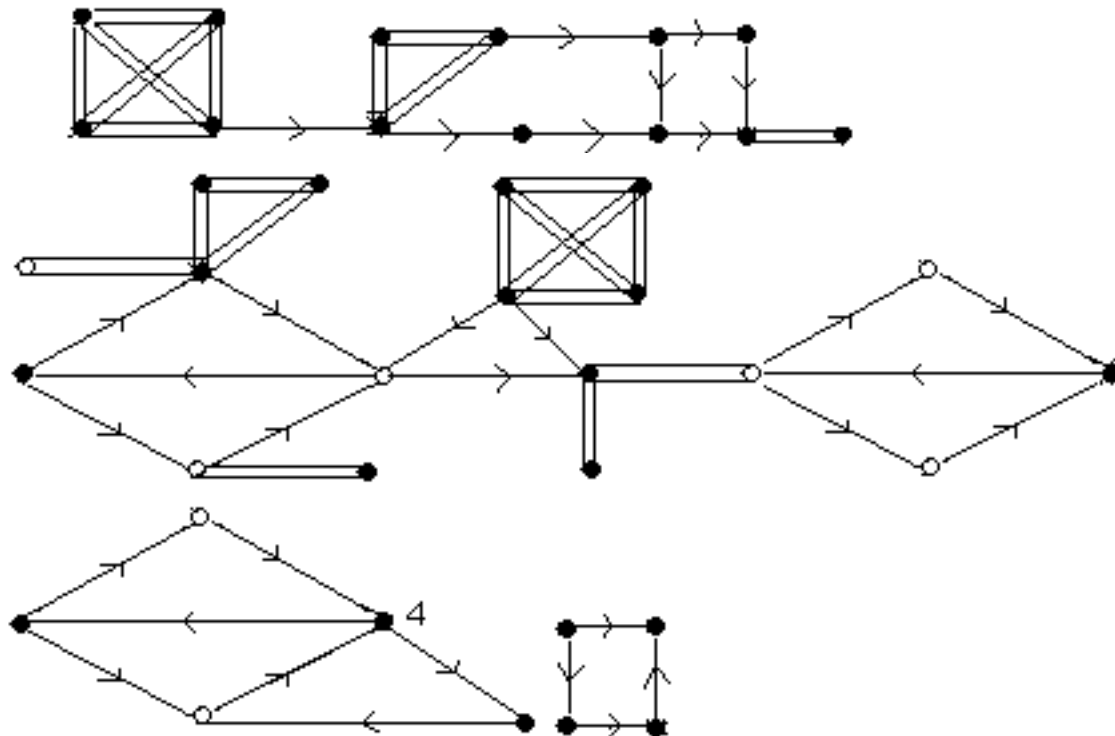
Definitions

- The matrix A is an M-matrix if and only if every off-diagonal entry of A is nonpositive and A is positive stable if and only if every off-diagonal entry of A is nonpositive and every principal minor of A is positive [HJ].
- The partial matrix B is a partial M-matrix if and only if every fully specified principal submatrix of B is an M-matrix and every specified off-diagonal entry of B is nonpositive.

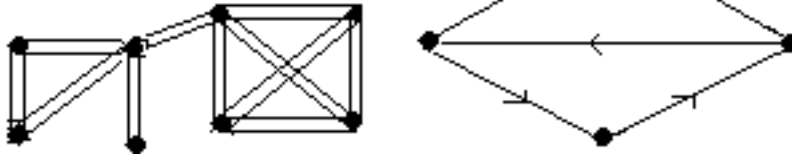
Results:

- A partial M-matrix that includes all diagonal entries can be completed to an M-matrix if and only if the zero completion (obtained by setting all unspecified off-diagonal entries to 0) is an M-matrix [JS1].
- A pattern Q (not necessarily positionally symmetric) has M-completion if and only if the principal subpattern determined by the diagonal positions of Q has M-completion. A pattern that includes all diagonal positions has M-completion if and only if every strongly connected induced subdigraph of its pattern-digraph is a clique [H2].

Examples: Have M-completion



Examples: Do not have M-completion



References:

- [H2] L. Hogben, Completions of M-matrix patterns. *Linear Algebra and Its Applications* 285 (1998), 143-152.
- [HJ] R. Horn and C. R. Johnson, *Topics in Matrix Analysis*, Cambridge University Press, Cambridge, 1991.
- [JS1] C. R. Johnson and R. L. Smith, The Completion Problem for M-Matrices and Inverse M-matrices, *Linear Algebra and Its Applications* 241-243 (1996), 655-667.