

## Class: Completely Positive Matrices (CP-matrices)

Note: Since this class is symmetric, all patterns are positionally symmetric and diagrams are graphs rather than digraphs.

Status: Done

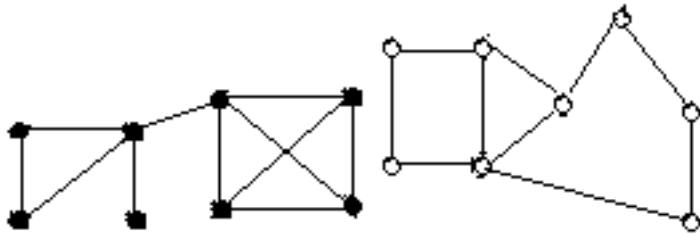
### Definitions

- The matrix  $A$  is a completely positive matrix (CP-matrix) if and only if  $A = CC^T$  for some nonnegative  $n \times m$  matrix  $C$ .
- The partial matrix  $B$  is a partial CP-matrix if and only if every fully specified principal submatrix of  $B$  is a CP-matrix, and whenever  $b_{ij}$  is specified then so is  $b_{ji}$  and  $b_{ji} = b_{ij}$ , and all specified off-diagonal entries are nonnegative.

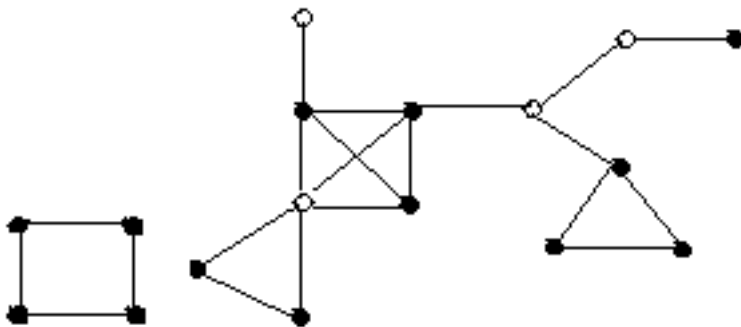
### Results:

- A pattern that includes all diagonal positions has CP-completion if and only if its pattern-graph is block-clique [DJ].
- A pattern has CP-completion if and only if every principal subpattern corresponding to a component  $H$  of its pattern-graph either omits all diagonal positions, or includes all diagonal positions and  $H$  is block-clique. [H5].

### Examples: Have CP-completion



### Examples: Do not have CP-completion



### References:

- [DJ] J. H. Drew and C. R. Johnson, The completely positive and doubly nonnegative completion problems, *Linear and Multilinear Algebra* 44 (1998), 85-92.
- [H5] L. Hogben, The symmetric M-matrix and symmetric inverse M-matrix completion problems, *Linear Algebra and Its Applications* 353 (2002) 159-168, draft available electronically in PDF format at <http://www.math.iastate.edu/lhogben/research/SymMatrixCompletion.pdf>.