1. Suppose $A, B \in \mathbb{C}^{n \times n}$ are idempotent. Prove $A + B$ is idempotent if and only if $AB = BA = 0$

2. Suppose $A, B \in \mathbb{F}^{n \times n}$ are commuting idempotent matrices. Prove that

$$\text{range}(AB) = \text{range} A \cap \text{range} B.$$ 

3. Suppose $A, B \in \mathbb{C}^{n \times n}$, $AB = B$ and $BA = A$.

   a) Prove $A$ and $B$ are idempotent.
   
   b) Either prove $A = B$ or give a counterexample to show this is false.

4. Suppose $A \in \mathbb{C}^{n \times n}$, for all $\lambda \in \text{spec}(A)$, $|\lambda| = 1$, and for all $v \in \mathbb{C}^n$, $|\langle Av, v \rangle| \leq ||v||^2$. Prove $A$ is unitary.

5. Suppose $A, B \in \mathbb{C}^{n \times n}$ are normal and $p_A(x) = p_B(x)$. Prove that $A$ and $B$ are similar.