Directions: Because some of this homework covers undergraduate linear algebra that has not been carefully developed in this class, it may be unclear what results you can use. Thus an assortment is listed on the next page that I think is sufficient. If you have another result you want to use, ask Leslie by e-mail it may be allowed (the reason for e-mail is so that there is a written record). You may also use any result covered in the text sections 1.1, 1.2, 1.3, 2.1, 2.2 or in class.

1. Let \( M = \begin{bmatrix} I_s & A \\ B & I_s \end{bmatrix} \in F^{2s \times 2s} \). Show \( \text{rank } M = s \) if and only if \( B = A^{-1} \).

2. Problem 21 in Section 1.2 (omit last question “When is \( C \) invertible?”).

3. Let \( A, B \in F^{n \times n} \).

   (a) Show that \( \begin{bmatrix} A & B \\ B & A \end{bmatrix} \) is similar to \( \begin{bmatrix} A - B & B \\ 0 & A + B \end{bmatrix} \).

   (b) Show that \( \det \begin{bmatrix} A & B \\ B & A \end{bmatrix} = \det(A + B) \det(A - B) \).

4. Let \( A \in C^{n \times n} \) and let \( \lambda \in \text{spec}(A) \). Prove that \( A \) is similar to a matrix having the sum of the elements in each row equal to \( \lambda \).

5. Problem 14 in Section 1.2. [Hint: You may want to use results from a later section in Chapter 1 and/or undergraduate material.]
Basic linear algebra results that may be assumed:

- High school algebra and properties of real and complex numbers.
- Anything in Sections 1.1, 1.2, 1.3 except the problems.
- Basic arithmetic of matrices, including $A0 = 0$ and if $O$ represents zero matrices of appropriate size, $OA = O = AO$.
- Principle of induction.
- $(A + B)^* = A^* + B^*$, $(AB)^* = B^*A^*$, $(cA)^* = \bar{c}A^*$.
- $\dim \text{range } A = \text{rank } A$.
- Dimension Theorem:
  If $A \in F^{m \times n}$, then rank $A + \text{null } A = n$.
  If $T: V \to W$ is linear and $V$ is finite dimensional, then $\dim \text{range } T + \dim \text{ker } T = \dim V$.
- Basic properties if eigenvalues/eigenvectors. For $A \in F^{n \times n}$ this includes:
  - $A$ is invertible if and only if $0 \notin \text{spec}(A)$. In this case, $\text{spec}(A^{-1}) = \{\frac{1}{\lambda} : \lambda \in \text{spec}(A)\}$.
  - $\text{spec}(A + aI) = \text{spec}(A) + a$.
- $\mathbb{C}^n$ is an inner product space with $(v, w) = w^*v$. 

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