Runge–Kutta–Fehlberg method

In mathematics, the Runge–Kutta–Fehlberg method (or Fehlberg method) is an algorithm in numerical analysis for the numerical solution of ordinary differential equations. It was developed by the German mathematician Erwin Fehlberg and is based on the large class of Runge–Kutta methods.

The novelty of Fehlberg's method is that it is an embedded method from the Runge-Kutta family, meaning that identical function evaluations are used in conjunction with each other to create methods of varying order and similar error constants. The method presented in Fehlberg's 1969 paper has been dubbed the RKF45 method, and is a method of order $O(h^4)$ with an error estimator of order $O(h^5)$. By performing one extra calculation, the error in the solution can be estimated and controlled by using the higher-order embedded method that allows for an adaptive stepsize to be determined automatically.

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**Butcher tableau for Fehlberg's 4(5) method**

Any Runge-Kutta method is uniquely identified by its Butcher tableau. The embedded pair proposed by Fehlberg[2]

0
1/4  1/4
3/8  3/32  9/32
12/13 1932/2197  -7200/2197  7296/2197
1  439/216  -8  3680/513  -845/4104
1/2  -8/27  2  -3544/2565  1859/4104  -11/40
16/135  0  6656/12825  28561/56430  -9/50  2/55
25/216  0  1408/2565  2197/4104  -1/5  0

The first row of coefficients at the bottom of the table gives the fifth-order accurate method, and the second row gives the fourth-order accurate method.

**See also**
Notes

1. According to Hairer et al. (1993, §II.4), the method was originally proposed in Fehlberg (1969); Fehlberg (1970) is an extract of the latter publication.

References


External links


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