For each problem that uses Matlab, you should hand in a printout of the relevant script or function file(s), or a transcript of your interactive session (use the `diary` feature), plus whatever outputs or plots are requested. Put the problems in the proper order, and label all printouts clearly. The final output should have full accuracy (`format long`); intermediate results can be shorter, if you want.

1. (Faires/Burden 3.5 #20) It is suspected that the high amounts of tannin in mature oak leaves inhibit the growth of the winter moth (*Operophtera brumata L., Geometridae*) larvae that extensively damage those leaves in certain years. The following table lists the average weight of two samples of larvae at times in the first 28 days after birth. The first sample was reared on young oak leaves, whereas the second sample was reared on mature leaves from the same tree.

You can find this data already typed in, in file `moth.txt`.

(a) Use a not-a-knot cubic spline to approximate the average weight curve for each sample. Plot the result for `t = 0:0.1:28`, with circles around the original data points.

(b) Repeat part (a) with cubic Hermite splines.

(c) (no credit for this part) Comment on the appearance of the graphs. Do they look reasonable? What do the graphs tell you about the original question (does the tannin inhibit growth)?

(d) (2 points extra credit) The original problem actually says "use a free spline." Figure out how to do that in Matlab, and repeat the problem with a free spline. This will require the spline toolbox.

Day	0	6	10	13	17	20	28
Sample 1	6.67	17.33	42.67	37.33	30.10	29.31	28.74
Sample 2	6.67	16.11	18.89	15.00	10.56	9.44	8.89

(10)

2. **(a)** Find the Padé approximation around $x = 0$ with $m = n = 2$ for $f(x) = \tan x$. The tangent function has a singularity at $x = \pm\pi/2$. Where does the Padé approximation have its singularity?

Plot the tangent function and its Padé approximant in steps of 0.1 around the origin (in both directions) until the functions reach their first singularity. Plot both functions in the same picture. Use the `axis` command to truncate the y-axis at ± 10.

(b) Repeat part (a) for $m = n = 3$.

Hint: The symbolic toolbox in Matlab is very handy, especially the `taylor` and `coeffs` functions. (15)

3. (Faires/Burden 8.4 #5d) Using Chebyshev polynomials, find the best continuous least squares approximation by a polynomial of degree 4 to the function $f(x) = x \ln x$ on the interval $[1, 3]$.

The resulting polynomial can be written as a sum of Chebyshev polynomials. Print out those coefficients.

Plot both functions in one picture, and also plot the error in a separate picture.

Hints: Chebyshev polynomials are defined on $[-1, 1]$. The given problem is equivalent to approximating $(x + 2) \ln(x + 2)$ on $[-1, 1]$.

The first four Chebyshev polynomials are listed in the book. Don't forget that they are not normalized. Also, don't forget the weight function when you do the inner products.

The Matlab symbolic toolbox can do some of the integrals required in closed form, but not all. You can use `double(int(...))` to force the symbolic toolbox to give you a numerical value. Ignore the warning messages. I would not recommend the `quad` function; it is not suited to the singular integrals you have to do here.

The approximating polynomial will be very close to the function. If yours is not, you are making a mistake. (15)

4. (a) The file `elnino.txt` contains monthly averages of the Southern Oscillation Index (related to El Niño) for the years 1962 through 1975. The SOI represents the difference in atmospheric pressure between Easter Island and Darwin, Australia.

Compute the Discrete Fourier Transform of this data, plot the absolute value, and identify the location of the three largest peaks.

Notes: The peak at 0 does not count. That is just the average value. The second half of the FFT is a mirror image of the first, so only look in the first half for the peaks.

(b) Which frequencies do these peaks correspond to?

Hint: In the Discrete Fourier Transform, the kth Fourier coefficient corresponds to the sine/cosine with k full periods inside the data interval. Remember the numbering starts with 0.

You may want to go through `showdemo sunspots` in Matlab. (10)