Math 481 HOMEWORK 2 due Thursday, Feb 26, 2009
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For each problem that uses Matlab, you should hand in a printout of the relevant script or function file(s), or a transcript of your interactive session (use the `diary` feature), plus whatever outputs or plots are requested. Put the problems in the proper order, and label all printouts clearly. The final output should have full accuracy (`format long`); intermediate results can be shorter, if you want.

1. You are given interpolation data

x	0	1	3	4	5
$f(x)$	1	1	2	2	3

(a) Build a divided difference table and find the interpolating polynomial $p(x)$ in Newton form. Evaluate it at the point $x = 2$.

(b) Write out the Lagrange interpolating polynomial $L_{4,1}(x)$ for this data. You don't have to multiply it out. (Note: the points are numbered x_0 through x_4). (15)

2. Assume that p_5 is the interpolating polynomial for the function e^x at the points $0, 1, 2, 3, 4, 5$. Give upper and lower bounds for the interpolation error at $x = 2.5$ and $x = -2.5$.

Hint: You don't have to find the interpolating polynomial to do this. (10)

3. Find the polynomial of degree 4 which solves the generalized interpolation problem

x	0	1	2
$p(x)$	2	-4	44
$p'(x)$	-9	4	

(a) By setting up and solving a system of equations

(b) Using a divided difference table (15)

4. Interpolate the function

$$f(x) = \frac{1}{1 + x^2}$$

at the points $x_0 = -5$, $x_1 = -4.5$, $\ldots$, $x_{20} = 5$ by a polynomial of degree 20. This is known as the *Runge example*. Use the Matlab built-in function `polyfit`.

For x going from -5 to 5 in steps of 0.1, plot the original f and the interpolating polynomial. Mark the interpolation points with circles. Set the y-axis limits to -0.5 and 1.5, so that $f(x)$ is nicely scaled. Parts of the polynomial will be cut off.

What is the largest error at the points where you evaluated $f(x)$?

Don't print all the values. You don't even have to print the polynomial or spline coefficients. Just show me the Matlab code you used, and the pictures. (10)