SOME SOLUTIONS TO THE FIRST EXAM, MATH 201, SPRING 08

1. a. We say $a|b$ provided there is an integer s such that $b = as$. (Any letter can be used in place of s.)
   b. We say $c \equiv d(\text{mod } m)$ provided that $m|(c - d)$.
2. a. For each real number s there exists a real number t such that $t^2 > s$. ("Every" can be used in place of "each." "All" is not recommended with the quantifier sequence $\forall\exists$ because it can be confusing, but I gave credit for it.)
   b. In symbols: $(\exists s \in \mathbb{R})(\forall t \in \mathbb{R})(t^2 \leq s)$. In English: There exists a real number s such that for all real numbers t, $t^2 \leq s$.
3. a. True (because the hypothesis is false, and $p \implies q$ is true when p is false regardless of whether q is true or false).
   b. $2 + 2 = 5$ and $3 + 4 \neq 7$ and $1 + 3 \neq 9$. (The sentence should really start with a word, but I did not grade on that. "It is the case that" would work.)
   c. If $(3 + 4 = 7$ or $1 + 3 = 9)$ then $2 + 2 = 5$.
   d. If $(3 + 4 \neq 7$ and $1 + 3 \neq 9)$ then $2 + 2 \neq 5$. (Notice the "and".)
4. The two important columns should read TFTFTFTT if the standard ordering of the lines is used.
5. a. $\{1, 2, 3, 4, 5\}$
   b. $\{3, 5\}$
   c. $\{1, 4\}$
   d. $\{2, 3, 5, 6\}$. Notice that 2 and 6 are in the set because they make the hypothesis $x \in A$ false, and therefore the implication is true.