## ASSIGNMENT 33 (ASSIGNED 4/23, DUE 4/25)

**Proposition 10:** The function defined by "$f(x) = 0$ if $x$ is rational and $f(x) = 1$ if $x$ is irrational" is not Riemann integrable on any interval $[a, b]$. (Remark: There is another approach to integration, the Lebesgue integral, which is stronger than the Riemann integral. This function is Lebesgue integrable, but there are others that are not. Lebesgue integration is usually studied in graduate school.)

**Proposition 11:** Let $f$ be a bounded function defined on $[a, b]$. Then $f$ is Riemann integrable if and only if for every $\varepsilon > 0$ there exists a partition $P$ such that $U(P, f) - L(P, f) < \varepsilon$.

**Assignment**

1. Prove Proposition 10 (done in class without notes).
2. Prove Proposition 11 (done in class without notes).