

ASSIGNMENT 25 (ASSIGNED 3/28, DUE 3/31)

Reminder: If you miss a class when a proof is done without notes, you must come to office hours or make an appointment to see what you have missed. You must also get notes on the proofs that were done in class with notes, since you are responsible for these on exams.

We will have an exam on April 7. (See the main web page.)

1. Prove that if f is differentiable at a then f is continuous at a . (The proof will be discussed in class, without notes.)
2. For each natural number n , f_n be the n -th power function, defined by $f_n(x) = x^n$. Prove by induction on n that for all real numbers a , $f'_n(a) = na^{n-1}$. It is understood here that $x^0 = 1$, so that for $n = 1$ the claim is $f'_1(a) = 1$. (This theorem is often written in the form

$$\frac{d}{dx}x^n = nx^{n-1}.$$

But do not use this notation in your proof.) Hints: As usual for induction, let P_n be the statement to be proved. Prove P_1 using the definition of derivative. Then assume P_k and prove P_{k+1} using the product rule, which we will state and prove in class. (Although P_2 has already been proved from the definition of derivative in the previous assignment, do not use that method again to prove P_2 this time.)