ASSIGNMENT 24 (ASSIGNED 3/26, DUE 3/28)

Definition: The function f is **continuous at** a if

$$\lim_{x \to a} f(x) = f(a).$$

Theorem: Any polynomial function is continuous. A rational function (quotient or ratio of polynomials) is continuous at points where the denominator is not zero.

Proof of this theorem was sketched in class.

Definition: The function f is **differentiable at** a if $\lim_{x \to a}(f(x) - f(a))/(x - a)$ exists. In this case the value of the limit is called the **derivative** of f at a and is denoted $f'(a)$. This may be written

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a},$$

with the proviso that the left hand side is not defined if the limit on the right hand side does not exist.

Assignment:

1. Prove that if f is the function defined by $f(x) = x^2$ for all $x \in \mathbb{R}$, and if $a \in \mathbb{R}$, then $f'(a) = 2a$. Be careful to make only correct statements in your proof, and to avoid common "sloppy" statements that are "good enough" sometimes, but not exactly correct. (This will be explained in class.)
2. Prove that the function $f(x) = |x|$ is continuous at 0 but is not differentiable at 0. (The proof will be sketched in class.)