ERRATA: “MODES,”
ROMANOWSKA & SMITH

Line 38 + 4: There is a function $f : X \cup \Omega \to \mathcal{P}(X)$ with $xf = \{x\}$ for x in X and $\omega f = \varnothing$ for ω in Ω. By the universality property (1.4.1) for $(X \cup \Omega)^*$, the mapping f can be uniquely

Line 39 − 10: $\omega \mapsto \omega\tau$

Line 52 + 8: Lemma 1.5.10

Line 53 + 9: is satisfied in all rectangular bands

Line 53 − 10: holds in all normal bands

Line 63 − 7: Hence $B = p + W$.

Line 64 − 1: operations P and

Line 67 + 2: **then** c **else** a

Line 97 − 5: $j_1 + \cdots + j_n$ (not j_r)

Line 97 − 4: Delete first sentence.

Line 97 − 3: $k_1 + \cdots + k_n$

Line 104 + 3: $0 \leq x < 1$

Line 106 − 7: $A \leq_s \prod_{i \in I} A_i \Leftrightarrow \bigcap_{i \in I} \ker \pi_i = \widehat{A}$ and each π_i surjects.

Line 106 − 4: $\bigcap_{i \in I} \theta_i = \widehat{A}$ and $\forall\, i \in I\,,\ A_i \cong A^{\theta_i}$

Line 111 + 7: $\bigcap_{i \in I} \theta_i = \widehat{A}$ and $\forall\, i \in I\,,\ A_i \cong A^{\theta_i}$

Line 134 + 12: iff $\mathfrak{P}K \models u = v$.

Line 145 + 6: (c) $((u,v) \in \Sigma$ *and* $(v,w) \in \Sigma) \Rightarrow ((v,w) \in \Sigma)$;

Line 146 + 3: Define $J : B \to \mathfrak{P}$

Line 149 + 6: *and* $(v,w) \in \Sigma)$

Line 157 + 10: $\varphi_{k_i,k}$

Line 158 + 8:

$$\begin{tikzcd} A \\ A_l \arrow[u, "\varphi_l"] \end{tikzcd}$$

Line 159 + 13: $h_k : X\Omega^{\varphi} \to A_k$

Line 166 − 1: $a \notin U$.

Line 168 + 8: Consequently $A/U =$

Line 171 + 2: $\subseteq DP\underline{\underline{K}}$

Line 171 − 4: $\&_{m \in i}$

Line 172 + 12: $\{X \in \mathcal{P}_{<\infty}(M) \mid$

Line 172 + 15: $\mathcal{P}_{<\infty}(M)$

Line 174 − 5: Exercise 3.7R

Line 175 − 5:

$$h_i : \prod_{i\in I}$$

Lines 186 + 1 to +4: Replace **Example 4.1.10** as follows:

> **Example 4.1.10.** In the free semigroup X^+ (cf. Section 1.4) over $X = \{x, y\}$, one has $x^2 \preceq x^2y$ and $y^2 \preceq xy^3$, but it is not true that $x^2y^2 \preceq x^2yxy^3$. It follows that this algebra is not naturally quasi-ordered. ☐

Line 217 + 8: *fibre* (B_{i^α}, Ω)
Line 218 − 7: *a subdirect product of* (I, Ω) *and the envelopes* (E_i, Ω) *for* $i \in I$,
Line 218 − 6: $\leq_s \prod_{i\in I}(E_i, \Omega) \times (I, \Omega)$
Line 218 − 3: It follows that $\mu \cap \ker \pi = \widehat{A}$.
Line 218 − 2: all the $(A, \Omega)^{\mu_i}$ and (I, Ω).
Line 219 + 3: *of the algebras* (I, Ω) *and* (C_i, Ω) *for* $i \in I$,
Line 219 + 6:

$$(A, \Omega) \leq_s \prod_{i\in I}(C_i, \Omega) \times (I, \Omega).$$

Line 219 − 11: Lemma 4.5.9
Line 219 − 10: It follows that $\mu \cap \ker \pi = \widehat{A}$.
Line 219 − 9: the $(A, \Omega)^{\mu_j}$ and $\ker \pi$.
Line 219 − 8: Lemma 4.5.9
Line 221 + 4: all the (Ci, Ω) and (I, Ω).
Line 221 + 6: the (Ci, Ω) or (I, Ω).
Line 224 − 11, 10: Corollary 4.5.5
Line 224 − 10: Theorem 4.5.3
Lines 226 + 7 to − 8: Replace text as follows:

Let

$$P_j^E := \bigcup(E_i \mid i \preceq j)$$

and

$$P_j := \bigcup(A_i \mid i \preceq j).$$

Obviously P_j^E is a subalgebra of (B, Ω) and a functorial sum of the E_i. The union P_j is a subalgebra of (P_j^E, Ω). Let $\mu(j)^E$ be the relation on P_j^E defined by

$$(a_k\psi_{k,r}, b_l\psi_{l,s}) \in \mu(j)^E \;:\Leftrightarrow$$
$$a_k\psi_{k,j} = a_k\psi_{k,r}\psi_{r,j} = b_l\psi_{l,s}\psi_{s,j} = b_l\psi_{l,j}$$

for $k \preceq r$, $l \preceq s$ and $r, s \preceq j$. It is easy to see that it coincides with the relation δ of Proposition 4.2.7. Hence it is a congruence relation of (P_j^E, Ω), and $(P_j^E, \Omega)^{\mu(j)^E} \cong E_j$. Now let $\mu(j)$ be the relation $\mu(j)^E$ restricted to P_j, i.e.

$$(a_k, b_l) \in \mu(j) :\Leftrightarrow a_k\psi_{k,j} = b_l\psi_{l,j}.$$

Then by the Second Isomorphism Theorem 1.2.4, $\mu(j)$ is a congruence of P_j. It obviously preserves (A_j, Ω), and $(P_j, \Omega)^{\mu(j)} \cong E_j$ [Exercise 4.5I]. Note also that

$$(a_k\psi_{k,r}, b_l\psi_{l,s}) \in \mu(j)^E \text{ if and only if } (a_k, b_l) \in \mu(j).$$

We will show that (E_j, Ω) is an envelope of (A_j, Ω). Let $\lambda \geq \mu(j)$ be a congruence on (P_j, Ω) preserving (A_j, Ω). For $k, l \preceq j$, let $(a_k, b_l) \in \lambda$. Since $(a_k\psi_{k,j}, a_k) \in \mu(j)^E$ and $(b_l\psi_{l,j}, b_l) \in \mu(j)^E$, it follows that $(a_k\psi_{k,j}, b_l\psi_{l,j}) \in \mu(j)^E$. Hence $(a_k, b_l) \in \mu(j)$. This implies that $\lambda \leq \mu(j)$, and consequently that $\lambda = \mu(j)$ is a maximal congruence preserving (A_j, Ω).

Line 227 + 4: $x\varphi_{j,k} = a_i\varphi_{i,j}\varphi_{j,k} := a_i\varphi_{i,k}$
Line 232 + 1: concrete strongly irregular varieties
Line 264 − 3: 5.5.2.2.
Line 271 + 1: for $x = a_1$,
Line 271 − 12: $R : (A, \cdot) \to (\mathrm{End}(A, \cdot), +)$;
Line 290 − 5:

$$= xp'q' + \Big(y\frac{pq'}{p+q-pq} + z\frac{q}{p+q-pq}\Big)(p+q-pq)$$

Line 290 − 2:

$$\frac{pq'}{(p'q')'} + \frac{q}{(p'q')'} = 1$$

Line 294 + 3: as a congruence of $(XC, \underline{I}^{\circ})$,
Line 294 + 4: subalgebra of $(XC \times XC, \underline{I}^{\circ})$
Line 300 − 5:

$$= xy(w_2 \cdot w_1).$$

Line 303 − 7:

$$\varphi : S \to X^{X \times X}$$

Line 304 + 12:

(d) Show that if there is a faithful action of a semigroup $(S, \cdot)$ on S satisfying

$$x\,yzs\,t = xyt\,xzt\,s \qquad \text{and}$$
$$xys\,uvs\,t = xut\,yvt\,s\,,$$

then $(S, \cdot)$ is commutative.

Line 368 − 6: (Cf. Exercise 6.4O.)

Line 377 − 11: quasi-identity (6.2.8)

Line 377 − 6: violates (6.2.8)

Line 377 − 3: condition (6.2.8)

Line 385 − 3: characterize certain Lallement

Line 386 + 5: let $\underline{\underline{\mathbb{I}}}$ be the quasivariety of Ω-semilattices.

Line 386 + 8: (E_j, Ω)

Line 386 + 9: $i \preceq j$

Line 386 − 1: $a_1 \dots a_{\omega\sigma-1} c_i a_{\omega\sigma+1} \dots a_n \omega$

Line 387 + 1: holds for each m-ary

Lines 390 − 12 to − 9: Replace text as follows:

We conclude this section with a version of Theorem 7.4.2 concerning the case where $\underline{\underline{\mathbb{I}}}$ is any irregular variety of modes. By Płonka's Theorem 4.3.2, the variety $\underline{\underline{\mathbb{I}}}$ is defined by a set of regular identities and an identity $x \cdot y = x$. Let (A, Ω) be a Lallement sum as in the introduction to Lemma 7.4.1. Now $\underline{\underline{\mathbb{I}}}$-algebras all have a full algebraic quasi-order. Thus for each $j \in I$, the sets P_j coincide with A and $E_j = A^{\mu(j)}$ for a maximal congruence μ_j of (A, Ω) preserving (A_j, Ω). Define a new relation μ_j on A as follows:

$$(b, c) \in \mu_j :\Leftrightarrow \forall a \in A_j, a \cdot b = a \cdot c.$$

Then a proof similar to that of Lemma 7.4.1 shows that the relation μ_j is the largest congruence on (A, Ω) preserving (A_j, Ω), and the envelope (E_j, Ω) is cancellative. This gives part of the proof of the version of Theorem 7.4.2 for the case where $\underline{\underline{\mathbb{I}}}$ is an irregular variety of modes. The last part of the proof of that version goes like the proof of Theorem 7.4.2. for the case where $\underline{\underline{\mathbb{I}}}$ is the quasivariety of Ω-semilattices.

We do not know if Theorem 7.4.2 holds also in the general case where $\underline{\underline{\mathbb{I}}}$ is any quasivariety of naturally quasi-ordered Ω-modes. Note that in this situation, we could not use the congruence μ as defined before Lemma 7.4.1 (or above). Indeed, the elements

$$a_1 \dots a_{\omega\sigma-1} b a_{\omega\sigma+1} \dots a_n \omega$$

and

$$a_1 \dots a_{\omega\sigma-1} c a_{\omega\sigma+1} \dots a_n \omega$$

in the second paragraph of the proof of Lemma 7.4.1 would not necessarily both be in A_j. A similar situation arises in the last paragraph of the proof of Lemma 7.4.1, and in the last paragraph of the proof of Theorem 7.4.2.

Line 390 − 8: Replace with:

> **Corollary 7.4.6.** Let $\underline{\underline{I}}$ be an irregular variety of modes.

Line 391 + 4: zero bands form an irregular variety, Corollary
Lines 391 + 7 to + 13: Replace **Example 7.4.8** as follows:

> **Example 7.4.8.** Consider the differential groupoid A given in the example following Proposition 5.7.8. It is a Lallement sum of two subgroupoids A_0 and A_1 with the envelopes $E_0 = \{0^{\mu(0)}, 1^{\mu(0)}, 0'^{\mu(0)}\}$ and $E_1 = \{0^{\mu(1)} = 1^{\mu(1)}, 0'^{\mu(1)}\}$ over the two element left-zero band $\{0, 1\}$ by the sum homomorphisms $0'\varphi_{1,0} = 0'^{\mu(0)}$ and $0\varphi_{0,1} = 1\varphi_{0,1} = 0^{\mu(1)}$ and $0'\varphi_{0,1} = 0'^{\mu(1)}$. Note that none of A_0, E_0 and E_1 is cancellative. Note also that $0 \preceq 1 \preceq 0$ and $0\varphi_{0,1} = 1\varphi_{0,1}$, but $0\varphi_{0,0} = 0 \neq 1 = 1\varphi_{0,0}$. It follows that it is not possible to extend the sum homomorphisms to functorial homomorphisms between E_i, or to embed A into a functorial sum of these envelopes.

Line 385 − 6: affine spaces over $\mathbb{D}$
Line 404 + 3: for $j < k$ and
Line 415 − 2: ω is an m-ary operation
Line 418 + 9: exists
Lines 427 + 8 to + 11: over an Ω-semilattice or an algebra in an irregular variety embeds as a subreduct into a functorial sum of affine spaces. We provide two theorems. One concerns sums over Ω-semilattices, while

Line 428 − 3: Theorem 4.5.18.
Line 429 − 10: $= c\psi_{j,k} \dots y\psi_{j,k} \dots e\psi_{j,k}\omega$
Lines 433 − 11 to − 10: and a variety $\underline{\underline{Q}}$ of Ω-semilattices or of irregular Ω-modes.
Line 433 − 8: By Theorem 7.4.2 and the remarks that follow it, the mode
Line 525 + 8: $L(p_x, \lambda, \gamma) =$
Lines 552 + 9, + 10: over its semilattice replica. Examples
Line 552 − 8: Note that the submodules of the module $(E, +, R)$ also form
Line 553 + 9: in AS, for
Line 554 + 5: A^r
Line 555 − 11: *variety of $\underline{\underline{V}}$-modals defined by a linear set of identities.*
Line 558 + 11: $F = \mathcal{A}(\_, \underline{T})$ and $G = \mathcal{X}(\_, \underset{\sim}{T})$

Line 558+12:

$$
\begin{array}{cccc}
A & & \mathcal{A}(A,\underline{T}) & f\alpha \\[2ex]
f\downarrow & \overset{\mapsto}{F} & \uparrow fF & \underline{\uparrow} \\[2ex]
B & & \mathcal{A}(B,\underline{T}) & \alpha
\end{array}
$$

$$
\begin{array}{cccc}
X & & \mathcal{X}(X,\underset{\sim}{T}) & g\alpha \\[2ex]
g\downarrow & \overset{\mapsto}{G} & \uparrow gG & \underline{\uparrow} \\[2ex]
Y & & \mathcal{X}(Y,\underset{\sim}{T}) & \alpha
\end{array}
$$

Line 561 + 4: tion of the variety of distributive lattices, using
Line 586 − 9: [1969] *Symmetric Spaces I: General Theory*, Benjamin, New York, New York.
Line 615 − 6: differential groupoid 269