(1) Let \( \text{Ob}(\text{Rel}) \) be the class of sets. For sets \( A \) and \( B \), let \( \text{Rel}(A, B) \) be the set of all subsets of \( A \times B \). For \( R \subseteq A \times B \) and \( S \subseteq B \times C \), let \( S \circ R = \{(a, c) \in A \times C \mid \exists b \in B . (a, b) \in R \text{ and } (b, c) \in S\} \). Show that \( \text{Rel} \) is a category.

(2) Let \( \text{Twoup} \) be the full subcategory of the category of sets comprising all sets with two or more elements. Show that \( \text{Twoup} \) has no initial object and no terminal object.

(3) Construe a non-trivial group as a category \( G \) with a single object, and with morphism set equal to the set of elements of the group. Show that the category \( G \) does not have products.

(4) Show that the poset category of divisors of 12 is isomorphic to a category of sets and functions.

(5) Suppose that a category \( C \) has a terminal object, and all pullbacks. Show that \( C \) has all equalizers.

(6) Let \( C \) be a category in which each morphism is a monomorphism, and for which there are two distinct morphisms having the same domain and the same codomain. Show that there are objects \( A \) and \( B \) of \( C \) for which the product \( A \times B \) does not exist.

(7) Let \( C \) be the category of complex vector spaces, and let \( R \) be the category of real vector spaces. Let \( G : C \to R \) be the forgetful functor that forgets the non-real scalar multiplications. Show that \( G \) has a left adjoint \( F \).

(8) Consider the functor \( S : \text{Set} \to \text{Set} \) with \( SA = A \times A \) and
\[
Sf : A \times A \to B \times B; (a, a') \mapsto (f(a), f(a'))
\]
for a function \( f : A \to B \). Show that \( S \) is naturally isomorphic to the functor \( \text{Set}(2, \_ \_ ) \), where \( 2 = \{0, 1\} \).

(9) In a category \( C \), an epimorphism \( u : B \to E \) is regular if it is the coequalizer of a pair of arrows \( f, g : A \to B \). Show that in the category \( \text{Set} \) of sets, each epimorphism is regular.

(10) Prove that the category of finite-dimensional real vector spaces is equivalent to the category \( \text{Matr}_\mathbb{R} \) of real matrices.