

## MATH 505 SPRING 2009 PRACTICE MID-TERM

*Hand in 3 answers for grading. Each question is worth  $8\frac{1}{3}$  points.*

- (1) Let  $S$  be a unital ring. Let  $V$  be a unital  $S$ -module such that a free  $S$ -module  $F$  is the direct sum  $U \oplus V$  of  $V$  with another  $S$ -module  $U$ . Show that  $V$  is projective.
- (2) For vector spaces  $U$  and  $V$  over a field  $K$ , prove that  $U \leq V$  and  $\dim U = \dim V < \infty$  imply  $U = V$ .
- (3) Let  $A$  be a commutative  $K$ -algebra. Suppose that  $(M, \cdot, 1)$  is a submonoid of  $(A, \cdot, 1)$ . Let  $\eta : A \rightarrow M^{-1}A$  be the canonical embedding. Show that  $\eta^{-1} : \text{Spec } M^{-1}A \rightarrow \text{Spec } A; P \mapsto \eta^{-1}(P)$  is injective, with image  $\{Q \in \text{Spec } A \mid Q \cap M = \emptyset\}$ .
- (4) Determine the invariant factors of the finite abelian group with multiset  $\langle 4\mathbb{Z}, 4\mathbb{Z}, 25\mathbb{Z}, 27\mathbb{Z}, 27\mathbb{Z}, 29\mathbb{Z} \rangle$  of elementary divisors.