(1) Prove or disprove the following statement for sets $A, B,$ and $C$:
If $A \cup B$ is isomorphic to $A \cup C$, then $B$ is isomorphic to $C$.
(2) Express the poset $(\{a,b,c\}, \subseteq)$ as an intersection of linear orders.
(3) Let $(X, G)$ be a transitive $G$-set for a group $G$. Prove that the following are equivalent:
(a) $\forall x \in X, \forall g \in G, xg = x \Rightarrow g = 1$;
(b) $\exists x \in X. \forall g \in G - \{1\}, xg \neq x$.
(4) Compute the size of each (group) conjugacy class of the group $A_5$.
(5) Let $e$ be an element of a group $(G, \cdot)$.
(a) Show that the set $G$ forms a group under the multiplication
$*: G \times G \to G; (x, y) \mapsto xe^{-1}y$.
(b) Show that the group $(G, *)$ with multiplication given in (a) is isomorphic with the original group structure $(G, \cdot)$ on $G$.
(6) Let $G$ be a 2-transitive permutation group on a set $X$ of size $n$. Prove that $|G|$ is divisible by $n(n - 1)/2$.
(7) Let $A, B,$ and $C$ be subgroups of a group $G$, with $A \subseteq C \subseteq AB$. Show that $C = A(B \cap C)$.
(8) Show that a group $G$ of order 135 cannot be simple. [You may use Sylov’s Theorems, if you carefully quote which ones you use.]