MATH 504 FALL 2008 PRACTICE FINAL

Each question is worth 6 points. The best 5 solutions will be taken.

(1) Prove or disprove the following statement for sets A, B, and C: If $A \cup B$ is isomorphic to $A \cup C$, then B is isomorphic to C.

(2) Express the poset $(2^{\{a,b,c\}}, \subseteq)$ as an intersection of linear orders.

(3) Let (X, G) be a transitive G-set for a group G. Prove that the following are equivalent:

  (a) $\forall x \in X$, $\forall g \in G$, $xg = x \Rightarrow g = 1$;

  (b) $\exists x \in X. \forall g \in G - \{1\}$, $xg \neq x$.

(4) Compute the size of each (group) conjugacy class of the group A_5.

(5) Let e be an element of a group $(G, \cdot)$.

  (a) Show that the set G forms a group under the multiplication

$$* : G \times G \to G; (x, y) \mapsto xe^{-1}y.$$

  (b) Show that the group $(G, *)$ with multiplication given in (a) is isomorphic with the original group structure $(G, \cdot)$ on G.

(6) Let G be a 2-transitive permutation group on a set X of size n. Prove that $|\,G\,|$ is divisible by $n(n-1)/2$.

(7) Let A, B, and C be subgroups of a group G, with $A \subseteq C \subseteq AB$. Show that $C = A(B \cap C)$.

(8) Show that a group G of order 135 cannot be simple. [You may use Sylov's Theorems, if you carefully quote which ones you use.]