MATH 307D FALL 2000 TEST #2

Write clearly. Box or underline your final answers to computational questions.
All questions carry equal weight.

1. Let \( A \) be the matrix
\[
\begin{bmatrix}
1 & 3 & 1 & -1 & 1 \\
0 & 0 & 1 & -1 & 1 \\
1 & 3 & 1 & 0 & 6
\end{bmatrix},
\]
(a) Find a basis for \( \text{Im} \, A \). (b) Find a basis for \( \text{Ker} \, A \).

2. For the matrix
\[
B = \begin{bmatrix}
1 & 0 & 1 \\
-1 & 1 & 0
\end{bmatrix},
\]
find all matrices \( A \) such that \( BA = I_2 \).

3. Suppose that the vectors \( u, v, w \) are linearly independent in \( \mathbb{R}^n \).
Show that the vectors \( u + v, v + w, w + u \) are also linearly independent.

4. For each of the following subsets of \( \mathbb{R}^2 \), decide whether or not the subset is a subspace. In each case, justify your answer.
(a) \( S = \{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x - y = 0 \} \).
(b) \( T = \{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x^2 + y^2 = 1 \} \).
(c) \( U = \{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x \geq y \} \).