MATH 307D FALL 2000 TEST #2

Write clearly. Box or underline your final answers to computational questions.
All questions carry equal weight.

1. Let A be the matrix
$$\begin{bmatrix} 1 & 3 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 1 & 3 & 1 & 0 & 6 \end{bmatrix}.$$
(a) Find a basis for $\operatorname{Im} A$. (b) Find a basis for $\operatorname{Ker} A$.

2. For the matrix
$$B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix},$$
find all matrices A such that $BA = I_2$.

3. Suppose that the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are linearly independent in $\mathbb{R}^n$. Show that the vectors $\mathbf{u} + \mathbf{v}, \mathbf{v} + \mathbf{w}, \mathbf{w} + \mathbf{u}$ are also linearly independent.

4. For each of the following subsets of $\mathbb{R}^2$, decide whether or not the subset is a subspace. In each case, justify your answer.
(a) $S = \{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x - y = 0 \}$.
(b) $T = \{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x^2 + y^2 = 1 \}$.
(c) $U = \{ \begin{bmatrix} x \\ y \end{bmatrix} \mid x \geq y \}$.