MATH 302 SPRING 2001 FINAL

Write clearly. Box or underline your final answers to computational questions. Each question is worth 5 points. Full credit is 40 points.

(1) Give a careful proof, quoting each ring axiom used at each step, that $(-1)^2 = 1$ in a ring R with multiplicative identity 1.

(2) Let $\mathcal{C}$ be a collection of ideals of a ring R. Show that the intersection $\bigcap_{J \in \mathcal{C}} J$ of the members of $\mathcal{C}$ is itself an ideal of R.

(3) Explain why $\mathbb{Z}_{11}$ cannot be an ordered field.

(4) A commutative ring K with multiplicative identity has no proper, non-trivial ideals. Show that K is a field.

(5) Give an example of a commutative ring R that is not a field, even though it has no proper, non-trivial ideals.

(6) Write down all 10 irreducible monic polynomials of degree 2 over $\mathbb{Z}_5$.

(7) The map $p : \mathbb{Z}_5 \to \mathbb{Z}_5; x \mapsto x^3 + 2$ is a permutation of $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$. Write the permutation p of $\mathbb{Z}_5$ as a product of disjoint cycles.

(8) The graphs of two cubic functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ each pass through the points $(0, 10)$, $(1, 6.5)$, $(2, 7.9)$ and $(3, 8.1)$. Prove that $f = g$.

(9) Explain why the fields $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ are not isomorphic.

(10) Explain why the fields $\mathbb{Q}(e)$ and $\mathbb{Q}(\pi)$ are isomorphic.

(11) Find a parity check matrix for the linear binary code of length 5 generated by the codewords 10111 and 11100.