

**MATH 301A SPRING 2009 PRACTICE TEST #2**

*Write clearly. Box or underline your final answers to computational questions.  
All questions carry equal weight.*

- (1) Find a solution  $x$  to the simultaneous congruences

$$x \equiv 3 \pmod{5},$$

$$x \equiv 7 \pmod{13}.$$

- (2) Let  $G$  be a finite group, with a subgroup  $S$  such that  $|G| = 2|S|$ . Show that  $S$  is a normal subgroup of  $G$ .
- (3) Give an example of a group  $G$ , and two subgroups  $S, T$  of  $G$ , for which  $ST$  is not a subgroup of  $G$ . Justify your answer.
- (4) Let  $x$  be an element of finite order in a group  $G$ . Show that the order of  $x^2$  is a divisor of the order of  $x$ .