1. (a) Determine the greatest common divisor \( d \) of 45 and 27.
(b) Express the greatest common divisor \( d \) as an integral linear combination of 45 and 27.

2. Let \( x_1, x_2, \ldots, x_{2m} \) be elements of a set \( X \). In the symmetric group \( X! \) on the set \( X \), show that
\[
\left( (x_1 \ x_2) \circ (x_3 \ x_4) \circ \cdots \circ (x_{2m-1} \ x_{2m}) \right)^{-1} = (x_{2m-1} \ x_{2m}) \circ \cdots \circ (x_3 \ x_4) \circ (x_1 \ x_2).
\]

3. For \( 0 < \theta < \pi/2 \), suppose that
\[
\cos \theta = \frac{l}{n} \quad \text{and} \quad \sin \theta = \frac{m}{n}
\]
with positive integers \( l, m, \) and \( n \). Show that at least one of \( l \) and \( m \) is even.

4. Consider the set
\[
G = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \middle| a, b, c \in \mathbb{Z}/5, ac = 1 \right\}
\]
of matrices over the ring of integers modulo 5.
(a) Show that \( G \) forms a group under multiplication.
(b) Show that \( |G| = 20 \).

5. Give an example of subgroups \( H \) and \( K \) of a group \( G \), such that \( HK \) is not a subgroup of \( G \). Explain why \( HK \) is not a subgroup.

6. Find a solution \( x \) to the simultaneous congruences
\[
x \equiv 3 \mod 5, \\
x \equiv 7 \mod 12.
\]

7. Let \( I \) and \( J \) be ideals of a ring \( R \), with \( I \) a subset of \( J \).
(a) Show that the quotient ring \( J/I \) is an ideal of \( R/I \).
(b) Show that the quotient rings \( R/J \) and \( (R/I)/(J/I) \) are isomorphic.