MATH 301 SPRING 2008 PRACTICE TEST #2

Write clearly. Box or underline your final answers to computational questions. All questions carry equal weight.

(1) Consider the set G of matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

satisfying the following three conditions:

(a) a, b, c, d are integers;

(b)

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = 1\,;$$

(c) a and d are odd, while b and c are even.

Determine whether G is or is not a subgroup of the general linear group $\mathrm{GL}(2, \mathbb{R})$ of all invertible matrices.

(2) Let $f : (X, \cdot, e_X) \to (Y, \cdot, e_Y)$ be a group homomorphism. Let N be a normal subgroup of X. Consider

$$f(N) = \{f(n) \mid n \text{ in } N\}\,.$$

(a) If $f : X \to Y$ is surjective, show that $f(N)$ is a normal subgroup of Y.

(b) Give a counterexample to show that $f(N)$ need not be a normal subgroup of Y, if $f : X \to Y$ is not surjective.

(3) Let H be a subgroup of a group G. Let N be a normal subgroup of G. Show that HN is a subgroup of G.

(4) Determine the group $(\mathbb{Z}/_{12})^*$ of units of the monoid $(\mathbb{Z}/_{12}, \cdot, 1)$ of integers modulo 12 under multiplication.