Let \( f : X \to Y; x \mapsto f(x) \) be a function.

(a) Show that there is a subset \( Y' \) of \( Y \) such that 
\[
g : X \to Y'; x \mapsto f(x)
\]
is surjective.

(b) Show that there is a subset \( X' \) of \( X \) such that 
\[
h : X' \to Y'; x \mapsto f(x)
\]
is bijective.

Write \( \sigma_a : \mathbb{R} \to \mathbb{R}; x \mapsto a + x \) for the shift by a real number \( a \).

Suppose that a group \( G \) of permutations of \( \mathbb{R} \) contains \( \sigma_a \) and \( \sigma_b \) for real numbers \( a \) and \( b \).

(a) Show that \( G \) contains \( \sigma_{ma} \) for each positive integer \( m \).

(b) Show that \( G \) contains \( \sigma_{ma} \) for each integer \( m \).

(c) Show that the group \( G \) contains \( \sigma_{ma + nb} \) for each integral linear combination \( ma + nb \) of \( a \) and \( b \).

Write each of the 10 symmetries of the regular pentagon

\[
\begin{array}{c}
1 \\
2 \\
3 \\
4 \\
0 \\
\end{array}
\]

in 3-space as a product of disjoint cycles.

Which of the following three conditions determines the kernel relation \( R \) of the cosine function \( \cos : \mathbb{R} \to \mathbb{R}; x \mapsto \cos x \)?

(a) \( x R y \iff x = \pm y \).

(b) \( x R y \iff x = 2\pi n \pm y \) for some integer \( n \).

(c) \( x R y \iff x - y = 2\pi n \) for some integer \( n \).