(1) Consider the numbers

\[1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10\].

Is it possible to put positive or negative signs in front of each, so that the total sum of the signed numbers is zero? Justify your answer.

(2) Consider an angle \(\theta\) with \(0 < \theta < \frac{\pi}{2}\). Suppose that

\[\cos \theta = \frac{l}{n} \quad \text{and} \quad \sin \theta = \frac{m}{n}\]

are rational numbers, with positive integers \(l, m,\) and \(n\).

(a) Give a justified example of an angle \(\theta\) with these properties.

(b) Show that \(l\) and \(m\) cannot both be odd numbers.

(3) Let \(\mathcal{P}_{\text{fin}}(\mathbb{N})\) denote the set of finite subsets of \(\mathbb{N}\).

(a) Show that \(\mathcal{P}_{\text{fin}}(\mathbb{N})\) forms a monoid under set union.

(b) Show that, under set intersection, \(\mathcal{P}_{\text{fin}}(\mathbb{N})\) does form a semigroup.

(c) Show that, under set intersection, \(\mathcal{P}_{\text{fin}}(\mathbb{N})\) does not form a monoid.

(4) Let \(u_1, u_2, \ldots, u_{r-1}, u_r\) be elements of a group \(G\). Give a careful proof, by induction on \(r\), that

\[(u_1u_2\ldots u_{r-1}u_r)^{-1} = u_r^{-1}u_{r-1}^{-1}\ldots u_2^{-1}u_1^{-1}\].