

MATH 301B FALL 2009 PRACTICE TEST #2

Write clearly, on separate paper. All questions carry equal weight. You will receive credit for your three best answers.

- (1) Let $d_k d_{k-1} \dots d_1 d_0$ be the decimal expansion of a positive integer n , so that

$$n = \sum_{j=0}^k d_j 10^j$$

with $0 \leq d_j < 10$. Show that 3 divides n if and only if

$$d_k + d_{k-1} + \dots + d_1 + d_0 \equiv 0 \pmod{3}.$$

- (2) Determine the group of units of the monoid of integers modulo 15 under multiplication.
- (3) Set

$$K = \left\{ \begin{bmatrix} k & k \\ k & k \end{bmatrix} \mid 0 \neq k \in \mathbb{R} \right\}.$$

- (a) Show that K is not a submonoid of the monoid $(\mathbb{R}_2^2, \cdot, I_2)$ of 2×2 real matrices under (the usual) matrix multiplication.
- (b) Show that K nevertheless forms a group under (the usual) matrix multiplication.
- (4) Let G be the group of orthogonal 2×2 real matrices under matrix multiplication. Let H be the subset of G comprising all orthogonal matrices of determinant 1.
- (a) Show that H is a subgroup of G .
- (b) For orthogonal matrices A and B , prove that $\det A = \det B$ if and only if $HA = HB$.