(1) Let \( d_k d_{k-1} \ldots d_1 d_0 \) be the decimal expansion of a positive integer \( n \), so that
\[
n = \sum_{j=0}^{k} d_j 10^j
\]
with \( 0 \leq d_j < 10 \). Show that 3 divides \( n \) if and only if
\[
d_k + d_{k-1} + \ldots + d_1 + d_0 \equiv 0 \pmod{3}.
\]

(2) Determine the group of units of the monoid of integers modulo 15 under multiplication.

(3) Set
\[
K = \left\{ \begin{bmatrix} k & k \\ k & k \end{bmatrix} \mid 0 \neq k \in \mathbb{R} \right\}.
\]

(a) Show that \( K \) is not a submonoid of the monoid \((\mathbb{R}^2, \cdot, I_2)\) of \(2 \times 2\) real matrices under (the usual) matrix multiplication.

(b) Show that \( K \) nevertheless forms a group under (the usual) matrix multiplication.

(4) Let \( G \) be the group of orthogonal \( 2 \times 2 \) real matrices under matrix multiplication. Let \( H \) be the subset of \( G \) comprising all orthogonal matrices of determinant 1.

(a) Show that \( H \) is a subgroup of \( G \).

(b) For orthogonal matrices \( A \) and \( B \), prove that \( \det A = \det B \) if and only if \( HA = HB \).