

MATH 301B FALL 2009 PRACTICE TEST #1

Write clearly. All questions carry equal weight. You will receive credit for your three best answers.

- (1) Let $f : X \rightarrow Y; x \mapsto f(x)$ be a function.
(a) Show that there is a subset Y' of Y such that

$$g : X \rightarrow Y'; x \mapsto f(x)$$

is surjective.

- (b) Show that there is a subset X' of X such that

$$h : X' \rightarrow Y'; x \mapsto f(x)$$

is bijective.

- (2) Let X be an infinite set. A function $f : X \rightarrow X$ is said to be *almost identical* if the set

$$\{x \in X \mid x \neq f(x)\}$$

of elements x of X , differing from their image $f(x)$ under f , is finite. Let F be the subset of X^X consisting of the almost identical functions. Show that F is a monoid of functions.

- (3) Write $\sigma_a : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto a + x$ for the shift by a real number a . Suppose that a group G of permutations of \mathbb{R} contains σ_a and σ_b for real numbers a and b .
(a) Show that G contains σ_{ma} for each positive integer m .
(b) Show that G contains σ_{ma} for each integer m .
(c) Show that the group G contains σ_{ma+nb} for each integral linear combination $ma + nb$ of a and b .

- (4) Let β , and $\alpha = (x_1 \ x_2 \ \dots \ x_{r-1} \ x_r)$, be permutations of a finite set X . Show that

$$\beta \circ \alpha \circ \beta^{-1} = (\beta(x_1) \ \beta(x_2) \ \dots \ \beta(x_{r-1}) \ \beta(x_r)).$$