1. (a) Determine the greatest common divisor $d$ of 45 and 27.
   (b) Express the greatest common divisor $d$ as an integral linear combination of 45 and 27.

2. Let $x_1, x_2, \ldots, x_{2m}$ be elements of a set $X$. In the symmetric group $X!$ on the set $X$, show that
   \[
   \left( (x_1 x_2) \circ (x_3 x_4) \circ \cdots \circ (x_{2m-1} x_{2m}) \right)^{-1} = (x_{2m-1} x_{2m}) \circ \cdots \circ (x_3 x_4) \circ (x_1 x_2).
   \]

3. For $0 < \theta < \pi/2$, suppose that
   \[
   \cos \theta = \frac{l}{n} \quad \text{and} \quad \sin \theta = \frac{m}{n}
   \]
   with positive integers $l, m, n$. Show that at least one of $l$ and $m$ is even.

4. Prove that $\log_{10} 7$ is irrational.

5. Let $M$ and $N$ be normal subgroups of a group $G$.
   (a) Show that the intersection $M \cap N$ is a normal subgroup of $G$.
   (b) Show that the quotient group $M/(M \cap N)$ is a normal subgroup of the quotient group $G/(M \cap N)$.

6. Let $X$ be a subset of a ring $R$. Show that
   \[
   C_R(X) = \{ r \in R \mid rx = xr \text{ for all } x \in X \}
   \]
   is a subring of $R$.

7. Let $I$ and $J$ be ideals of a ring $R$. Prove that $I + J$ is an ideal of $R$. 

Write clearly.
Box or underline your final answers to computational questions.
All questions carry equal weight.