

MATH 301C FALL 2007 PRACTICE FINAL

Write clearly.

Box or underline your final answers to computational questions.

All questions carry equal weight.

- Determine the greatest common divisor d of 45 and 27.
 - Express the greatest common divisor d as an integral linear combination of 45 and 27.
- Let x_1, x_2, \dots, x_{2m} be elements of a set X . In the symmetric group $X!$ on the set X , show that

$$\begin{aligned} & \left((x_1 \ x_2) \circ (x_3 \ x_4) \circ \cdots \circ (x_{2m-1} \ x_{2m}) \right)^{-1} \\ &= (x_{2m-1} \ x_{2m}) \circ \cdots \circ (x_3 \ x_4) \circ (x_1 \ x_2). \end{aligned}$$

- For $0 < \theta < \pi/2$, suppose that

$$\cos \theta = \frac{l}{n} \quad \text{and} \quad \sin \theta = \frac{m}{n}$$

with positive integers l, m, n . Show that at least one of l and m is even.

- Prove that $\log_{10} 7$ is irrational.
- Let M and N be normal subgroups of a group G .
 - Show that the intersection $M \cap N$ is a normal subgroup of G .
 - Show that the quotient group $M/(M \cap N)$ is a normal subgroup of the quotient group $G/(M \cap N)$.
- Let X be a subset of a ring R . Show that

$$C_R(X) = \{r \text{ in } R \mid rx = xr \text{ for all } x \text{ in } X\}$$

is a subring of R .

- Let I and J be ideals of a ring R . Prove that $I + J$ is an ideal of R .