

MATH 301C FALL 2007 PRACTICE TEST #1

Write clearly. All questions carry equal weight.

- (1) Show that $2n + 1$ and $5n + 3$ are coprime for all integers n .
- (2) Let $f : X \rightarrow Y$ be a function with nonempty domain X . Show that there is a function $g : Y \rightarrow X$ such that $f = f \circ g \circ f$.
- (3) Let S be the set of all affine functions

$$f : \mathbb{Z} \rightarrow \mathbb{Z}; x \mapsto mx + c$$

in which m is an odd integer and c is an even integer.

- (a) Show that S forms a monoid of functions on the set \mathbb{Z} of integers.
 - (b) Does S form a group of permutations on the set \mathbb{Z} ? Justify your answer.
- (4) Let β and

$$\alpha = (x_1 \ x_2 \ \dots \ x_{r-1} \ x_r)$$

be permutations of a finite set X . Show that

$$\beta \circ \alpha \circ \beta^{-1} = (\beta(x_1) \ \beta(x_2) \ \dots \ \beta(x_{r-1}) \ \beta(x_r)).$$