

## MATH 301A FALL 2006 PRACTICE FINAL

*Write clearly.*

*Box or underline your final answers to computational questions.*

*All questions carry equal weight.*

1. Find the greatest common divisor of 10 and 6. Express it as an integral linear combination of 10 and 6.

2. Let  $G$  be the set of all  $2 \times 2$  matrices of the form

$$\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

with real  $\theta$ . Show that  $G$  forms a group under matrix multiplication.

3. Show that each transposition  $(1\ s)$  in  $S_n$  (with  $1 < s \leq n$ ) is a product of transpositions from the set

$$\{(r\ r+1) \mid 1 \leq r < n\}.$$

4. Let  $J$  be the set of  $2 \times 2$  matrices whose entries are all even integers. Show that  $J$  is an ideal in the ring of all  $2 \times 2$  matrices over the ring of integers.

5. Let  $D$  be the set of all  $2 \times 2$  matrices of the form

$$\begin{bmatrix} x & -y \\ y & x \end{bmatrix}$$

with  $x$  and  $y$  from the ring  $\mathbb{Z}$  of integers. Show that  $D$  forms an integral domain.

6. Show that  $X^2 + X + 1$  is irreducible over  $\mathbb{Z}/5\mathbb{Z}$ .

7. Let  $J$  be the ideal  $(X^2 + X + 1)\mathbb{Z}_5[X]$  in the ring  $\mathbb{Z}_5[X]$  of polynomials over  $\mathbb{Z}_5$ . Find the multiplicative inverse of  $X + J$  in the field  $\mathbb{Z}_5[X]/J$ .