1. Find the greatest common divisor of 10 and 6. Express it as an integral linear combination of 10 and 6.

2. Let $G$ be the set of all $2 \times 2$ matrices of the form
   \[
   \begin{pmatrix}
   \cos \theta & \sin \theta \\
   -\sin \theta & \cos \theta
   \end{pmatrix}
   \]
   with real $\theta$. Show that $G$ forms a group under matrix multiplication.

3. Show that each transposition $(1 \ s)$ in $S_n$ (with $1 < s \leq n$) is a product of transpositions from the set
   \[
   \{(r \ r + 1) \mid 1 \leq r < n\}.
   \]

4. Let $J$ be the set of $2 \times 2$ matrices whose entries are all even integers. Show that $J$ is an ideal in the ring of all $2 \times 2$ matrices over the ring of integers.

5. Let $D$ be the set of all $2 \times 2$ matrices of the form
   \[
   \begin{pmatrix}
   x & -y \\
   y & x
   \end{pmatrix}
   \]
   with $x$ and $y$ from the ring $\mathbb{Z}$ of integers. Show that $D$ forms an integral domain.

6. Show that $X^2 + X + 1$ is irreducible over $\mathbb{Z}/5\mathbb{Z}$.

7. Let $J$ be the ideal $(X^2 + X + 1)\mathbb{Z}_5[X]$ in the ring $\mathbb{Z}_5[X]$ of polynomials over $\mathbb{Z}_5$. Find the multiplicative inverse of $X + J$ in the field $\mathbb{Z}_5[X]/J$. 