

## MATH 301A FALL 2006 PRACTICE TEST #2

*Write clearly. All questions carry equal weight.*

- (1) Let  $m$  and  $n$  be coprime integers. Show that the ring  $\mathbb{Z}/mn\mathbb{Z}$  is isomorphic to the ring  $\mathbb{Z}/m\mathbb{Z} \oplus \mathbb{Z}/n\mathbb{Z}$ .
- (2) Find a solution  $x$  to the simultaneous congruences
$$\begin{aligned}x &\equiv 3 \pmod{5}, \\x &\equiv 7 \pmod{12}.\end{aligned}$$
- (3)
  - (a) Show that  $S = \{0, 2, 4, 6, 8\}$  forms a subring of  $\mathbb{Z}/10\mathbb{Z}$ .
  - (b) Show that  $S$  forms a unital ring.
  - (c) Show that  $S$  is not a unital subring of  $\mathbb{Z}/10\mathbb{Z}$ .
- (4) Suppose that  $x^2 = x$  for each element  $x$  of a ring  $R$ . Show that  $R$  is commutative.