

MATH 301A FALL 2006 PRACTICE TEST #1

Write clearly. All questions carry equal weight.

- (1) Let β and

$$\alpha = (x_1 \ x_2 \ \dots \ x_{r-1} \ x_r)$$

be permutations of a finite set X . Show that

$$\beta \circ \alpha \circ \beta^{-1} = (\beta(x_1) \ \beta(x_2) \ \dots \ \beta(x_{r-1}) \ \beta(x_r)).$$

- (2) For each real number b and non-zero real number a , define

$$\alpha_{a,b} : \mathbb{R} \rightarrow \mathbb{R}; x \mapsto ax + b.$$

Show that $G = \{\alpha_{a,b} \mid b \in \mathbb{R}, 0 \neq a \in \mathbb{R}\}$ is a group under the operation of composition.

- (3) Let G be a finite group. Show that the equation

$$x^3 = e$$

has an odd number of solutions x in G .

- (4) Prove that $\sqrt[3]{2}$ is irrational.