Write clearly. All questions carry equal weight.

(1) Let $\beta$ and
$$\alpha = (x_1 \ x_2 \ \ldots \ x_{r-1} \ x_r)$$
be permutations of a finite set $X$. Show that
$$\beta \circ \alpha \circ \beta^{-1} = (\beta(x_1) \ \beta(x_2) \ \ldots \ \beta(x_{r-1}) \ \beta(x_r)).$$

(2) For each real number $b$ and non-zero real number $a$, define
$$\alpha_{a,b} : \mathbb{R} \rightarrow \mathbb{R}; \ x \mapsto ax + b.$$ 
Show that $G = \{\alpha_{a,b} \mid b \in \mathbb{R}, 0 \neq a \in \mathbb{R}\}$ is a group under the operation of composition.

(3) Let $G$ be a finite group. Show that the equation
$$x^3 = e$$
has an odd number of solutions $x$ in $G$.

(4) Prove that $\sqrt[3]{2}$ is irrational.