(1) Let $x$ and $y$ be integers. If $x + 5y$ is odd, prove that $x$ and $y$ have opposite parity.

(2) For an odd integer $n$, prove that $4 \mid n^3 + n^2 + n + 1$.

(3) Let $x$ and $y$ be positive real numbers. Prove that
$$\frac{x}{y} + \frac{y}{x} \geq 2.$$ 

(4) Let $A$, $B$ and $C$ be sets. Prove that
$$A \cup (B \cap C) = (A \cap B) \cup (A \cap C).$$