

## MATH 201B FALL 2009 PRACTICE FINAL

*Write clearly, on separate paper. All questions carry equal weight.  
You will receive credit for your five best answers.*

- (1) Let  $P$  and  $Q$  be statements. Show that

$$(P \rightarrow Q) \wedge (Q \rightarrow P) \equiv (P \vee Q) \rightarrow (P \wedge Q).$$

- (2) Let  $A$ ,  $B$ , and  $C$  be subsets of a set  $U$ . Prove that

$$(C \setminus A) \cup (A \setminus B) \cup (B \setminus C) = (A \cup B \cup C) \setminus (A \cap B \cap C).$$

- (3) Let  $n$  be an integer. Prove that  $5 \mid (n^4 - 1)$  implies  $5 \nmid n$ .

- (4) Let  $n$  be a positive integer. Prove or disprove:

$$3 \mid (2^{2n} - 1).$$

- (5) Prove that  $2^n > n^3$  for integers  $n \geq 10$ .

- (6) Give an  $\varepsilon$ - $\delta$  type proof that

$$\lim_{x \rightarrow 2} x^4 = 16.$$