MATH 201B FALL 2009 PRACTICE TEST #1

Write clearly, on separate paper. All questions carry equal weight. You will receive credit for your three best answers.

(1) Show that $\log_{\sqrt{2}} 5$ is irrational.

(2) Prove

$$\forall\ 0 < x \in \mathbb{R}\,,\ \forall\ 1 < n \in \mathbb{Z}\,,\ (1+x)^n > 1 + nx$$

by induction on n.

(3) Prove or disprove:

$$\forall\ n \in \mathbb{N}\,,\ 3^n > n^2\,.$$

(4) Prove or disprove:

$$\forall\ 1 < n \in \mathbb{Z}\,,\ 5 \,\Big|\, (4^n - 1)\ .$$