

Calculators are permitted for numerical calculations and for finding integrals.

Show all work.

1. (10 Points)

For the initial value problem (IVP)

$$t(t - 10)y'' + 3ty' + 4y = 2, \quad y(3) = 0, \quad y'(3) = -1$$

determine the largest interval in which this IVP is certain to have a unique solution.

2. (10 Points) A spring-mass system has a spring constant of 3 N/m. A mass of 2 kg is attached to the spring, and the motion takes place horizontally in a medium that imparts a viscous force of 2 N when the speed of the mass is 10cm/sec. Initially, the mass was pulled 8 cm away (the spring is stretched) from its equilibrium position and released.

a) State the Physics law to be used to set up the differential equation for the position function $x(t)$.

b) Set up the differential equation and briefly explain each term in the equation.

c) Set up the initial conditions.

(Do not solve the equation or the initial value problem.)

3. (20 Points) The homogeneous differential equation $y''' - y'' - y' + y = 0$ has a solution $y_1 = e^t$, $y_2 = te^t$ and $y_3 = e^{-t}$. Find the general solution to the nonhomogeneous equation $y''' - y'' - y' + y = -\cos t$.

4. (20 points) Solve the initial value problem $y'' - 2y' + 2y = 0$ $y(0) = 0$ $y'(0) = -1$.

5. (20 points) Find the general solution of the differential equation $y''' + y = 2e^{-t}$.
(Hint: $-1 = e^{i\pi+2k\pi}$ and $e^{i\theta} = \cos \theta + i \sin \theta$.)

6. (20 Points) a) Solve the homogeneous equation $y''' - y' = 0$.

b) Determine a suitable form for $Y(t)$ if the method of undetermined coefficients is used to solve the inhomogeneous equation $y''' - y' = \sin t + \cos(2t) + t^2e^t - 2e^{-t} + 4$. (*Do not* evaluate the coefficients.)