Math 266 (S. Hou) Sample Test 1 **Name**

Calculators are permitted for numerical calculations and for finding integrals; you will not get full credit for approximate answers found by calculators.
Show all work.

1. (15 Points) For each of the following differential equations, determine its order and state whether it is linear or nonlinear.

   a) $t^3\dfrac{d^2y}{dt^2} + (t+1)\dfrac{dy}{dt} = sint$

   b) $y'' + (e^t + 1)(y')^3 + ty^5 = 0$

   c) $y^{(4)} + y^6 + yy' = t^5 + 2t - 1$

2. (15 Points) Find the general solution of the linear equation $ty' + 2y = \sin t$, $t > 0$.

3. (15 points) Solve the initial value problem

$$y' = 2y^2 + xy^2, \qquad y(0) = 1.$$

4. (15 points) A tank initially contains 120 gallons of pure water. A brine with a 5 lb/gal salt concentration enters the tank at a rate of 3 gal/min, and the well-stirred mixture leaves the tank at a rate of 4 gal/min. Let $Q(t)$ denote the amount of salt in the tank at time t (in minutes). Set up a differential equation for Q and write out the initial condition. (Do not solve the differential equation.)

5. (20 points) Consider the autonomous equation $y' = (y-1)(y-2)$.

   a) Find all equilibrium solutions of this equation.

   b) Express y'' in terms of y and find all zeros of y''.

   c) Sketch the graphs of all solutions with initial value y_0 between 1 and 2; show all work determining the monotonicity and concavity of the solutions.

6. (20 Points) Given differential equation $(3x^2 - 2xy + 2)dx + (6y^2 - x^2 + 3)dy = 0$,

   a) verify that this equation is exact.

   b) find the solution of this differential equation satisfying the initial condition $y(0) = 1$.