

Math 267 SAMPLE EXAM 3**Name**

Calculators are permitted for numerical calculations, for finding integrals, for factoring polynomials, and for graphing functions; however, you will not get full credit for approximate answers found by calculators.

Show all work.

1. (15 Points) Find the Laplace transform of $f(t) = e^{2t}$ using the definition of the Laplace transform.
2. (20 Points) Express the following piecewise continuous function f in one formula and then find the Laplace transform of f :

$$f(t) = \begin{cases} e^t, & \text{if } 0 \leq t < 2 \\ 0, & \text{if } 2 \leq t < 4 \\ t^2, & \text{if } t \geq 4 \end{cases} .$$

3. (25 points)

a) Find the inverse Laplace transform of $G(s) = \frac{s^2 + 2s - 5}{(s - 1)^3}$

b) Let $g(t)$ be a piecewise continuous function whose Laplace transform $G(s)$ exists for $s > 0$. Express the inverse Laplace transform of $\frac{s}{s^2+4}G(s)$ in terms of a convolution integral.

4. (20 points) Solve the following initial value problem by the Laplace transform method

$$y'' + 2y' + 2y = 0 \quad y(0) = 1, \quad y'(0) = 0 .$$

5. (20 points)

Use the Laplace transform method to find the solution of the following initial value problem:

$$y^{(3)} - 2y'' + y' = \delta(t - 2) \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 0, \quad y'''(0) = 0 .$$