Math 655 Assignment #3

Problems listed below are related to
(a) Traveling waves
(b) Stationary phase method
(c) Homogenization

1. Consider
   \[ u_t = u_{xx} + u(1 - u^2). \]
   Find the traveling wave solution of the form
   \[ u = v(x - st), \]
   such that \( v(-\infty) = -1, \ v(+\infty) = 1 \) with proper choice of \( s \).

2. Let \( \phi \) and \( a \) be two given functions satisfying
   \[ D\phi(y_k) = 0, \ y_k \in suppt(a) \ k = 1, \cdots N, \]
   and \( D^2\phi(y_k) \) is non-singular for \( k = 1 \cdots N \).
   Find and justify the limit
   \[ \lim_{\epsilon \to 0} I_\epsilon, \ I_\epsilon := \int_{\mathbb{R}^n} a(y)e^{i\phi(y)/\epsilon}dy. \]

3. Let \( a(x, y) \) be a smooth, positive function and is 1-periodic in \( y \); \( f \in L^2(0, 1) \) is also given. Suppose that \( u^\epsilon \) solves the problem
   \[
   \begin{cases}
   -(a(x, \frac{y}{\epsilon})u^\epsilon_x)_x = f(x) & x \in (0, 1) \\
   u^\epsilon(0) = u^\epsilon(1) = 0.
   \end{cases}
   \]
   Show that \( u^\epsilon \rightharpoonup u \) weakly in \( H^1_0(0, 1) \), where \( u \) solves
   \[
   \begin{cases}
   -(\bar{a}(x)u_x)_x = f(x) & x \in (0, 1) \\
   u(0) = u(1) = 0,
   \end{cases}
   \]
   for \( \bar{a}(x) = (\int_0^1 a(x, y)^{-1}dy)^{-1} \).