Math 503 HW#5

More interpolation problems.

1. Using Stirling central difference interpolation formula to find the 1st (with \( \tilde{p} \in P_2 \)) and 2nd (with \( \tilde{p} \in P_4 \)) approximation to \( f''(x_k) \). Comment on their error bounds by using the Taylor expansion formula.

2. Let \( x_0, \ldots x_n \) be the zeros of the Legendre polynomial \( L_{n+1} \). Show:
   a) For every polynomial \( p \in P_n \)
      \[ \|p\|_2 \leq \sqrt{2} \max_{0 \leq k \leq n} |p(x_k)|. \]
   b) Let \( f \in C[-1, 1] \) and suppose \( \tilde{p}_n \in P_n \) are the polynomials which interpolate at \( x_0, \ldots x_n \). Then
      \[ \lim_{n \to \infty} \|f - \tilde{p}_n\|_2 = 0. \]

3. Program the complete Horner Algorithm, and for the polynomial \( p(x) = 3x^5 - 7x^4 + 2x^2 + 4x + 12 \), compute the coefficients of its expansion around the points \( \xi := 2, -1, -3 \) respectively.

4. Using the Neville’s Algorithm, find an approximation to \( e^{0.53} \), using the points \( x_k = 0.3 + kh \) with \( h = 0.1 \) for \( 0 \leq k \leq 5 \).

5. Approximate the function \( f(x) = \sin(\frac{\pi}{2} x) \) for \( x \in [0, 1] \) by simple cubic Hermite interpolation using the points \( x_0 = 0 \) and \( x_1 = 1 \). What is the maximum relative interpolation error in the intervals \([0.0.25],[0.25,0.75],[0.75,1]\).