MATH 517  Practice problems for system of conservation laws

Problems listed below are related to methods for solving system of conservation laws

\[ u_t + f(u)_x = 0 \]

We consider Euler equation of gas dynamics which is modeled by the above system with

\[
\begin{align*}
    u &= \left( \begin{array}{c} \rho \\ m \\ E \end{array} \right), \\
    f &= \left( \begin{array}{c} \rho v^2 + p \\ \rho v(E+p) \\ v(E+p) \end{array} \right), \\
    p &= (\gamma - 1)(E - 0.5\rho v^2), \\
    m &= \rho v
\end{align*}
\]

where $u$ has components of density, momentum and energy. Write a computer program using

(i) Lax-Friedrich scheme
(ii) Second order extension of (i) with minmod limiter

to test the following two benchmark problems for $\gamma = 1.4$:

a) The Lax problem with initial Riemann data at $x=0$

\[
    u_l = (0.445, 0.311, 8.928)^T, \quad u_r = (0.5, 1, 1.4275)^T.
\]

b) The Osher-Shu problem.

\[
(\rho, v, p)(x, 0) = \begin{cases} 
    (3.857143, 10.333333, 2.629369), & x < -0.8 \\
    (1 + 0.2\sin(5\pi x), 0.1), & x > -0.8 
\end{cases}
\]

For this example a density wave will emerge behind the shock discontinuity, its fine structure makes the current problem a suitable case for higher order methods.