MATH 517 HW #4

Problems listed below are related to methods for solving scalar conservation laws

\[ u_t + f(u)_x = 0 \]

(T1) Prove that any TVD scheme is monotonicity preserving.

(T2) Show that the Lax-Friedrichs scheme is \( l_1 \)-contracting provided the CFL condition \( \lambda |f'(u)| \leq 1 \) is satisfied.

(C1) Write a program to compute the Cauchy problem

\[ u_t + f(u)_x = 0, \quad u(x,0) = g(x) \]

where \( f \) is the flux function and \( g(x) \) the given initial data. Using

(1) The Godunov scheme
(2) The Lax-Friedrichs scheme
(3) The E-scheme
(4) The Lax-Wendroff scheme

(C2) Using the above program to test

(1) \( f = 0, \quad g(x) = \begin{cases} 1 & -0.5 < x < 0.5, \\ 0 \end{cases} \)

(2) \( f = u^2/2, \quad g(x) = 1/4 + \sin \pi x / 2 \) for \(-1 \leq x \leq 1.\)

(3) \( f = \frac{4u^2}{4u^2+(1-u)^2} \) and \( g(x) = \begin{cases} 1 & -0.5 < x < 0, \\ 0 \end{cases} \)

(4) \( f = \frac{1}{4}(u^2-1)(u^2-4) \) and \( g(x) = \begin{cases} 2 & x < 0, \\ -2 \end{cases} \) for \( x > 0.\)

Take say \( CFL = 0.5, \quad N = 100 \) and plot the solution at \( t = 0, \ 1, \ 2 \) followed by thoughtful comments on your numerical results.