Problem 8. There are 2012 points in the plane, no three of which are collinear. Is there always a circle that passes through at least three of the points and such that none of the 2012 points are inside the circle? If no give an example. If yes, give a proof.

Solution. Measure the distance between each pair of points, and let \( A \) and \( B \) be two points for which this distance is minimum. The circle \( C_0 \) with diameter \( AB \) contains none of the other 2010 points on the circle or in its interior. If \( X \) were a different point of the set and on or inside of \( C_0 \), then we would have \( AX < AB \), contradicting the choice of \( A \) and \( B \).

Label the line through the center of the circle and perpendicular to \( AB \) as the \( x \) axis and let 0 be the center of the circle. Note that any circle through \( A \) and \( B \) has its center on this \( x \)-axis. Let \( C_t \), with center \( t \), be the circle of minimal radius passing through \( A \), \( B \), and at least one of the other 2010 points. We claim that this circle does not have any of the other points of the set in its interior.

To prove this, assume that a point \( Y \) of the set of points is inside of \( C_t \). First note that \( Y \) and \( t \) cannot be on opposite sides of \( AB \) because then \( Y \) would be inside of \( C_0 \). Thus any such \( Y \) must be on the same side of \( AB \) as \( X \) (and outside of \( C_x \)). Without loss of generality \( Y \) is on the same side of line \( Ot \) as \( A \). Extend segment \( AY \) through \( Y \) to intersect \( C_t \) at \( Y' \). The perpendicular bisector of \( AY' \) intersects the \( x \)-axis in \( t \), so the perpendicular bisector of \( AY \) intersects the \( x \)-axis in a point \( t' \) between 0 and \( t \). But then \( C_{t'} \) is the circle through \( A \), \( B \), and \( Y' \) and has radius less than the radius of \( C_t \). However this contradicts the definition of \( C_t \), so we conclude that \( C_t \) passes through three of the 2010 points but does not have any of the 2010 points in its interior.