Problem 7. Given a positive integer \( n \), the number \( A_n \) consists of the following digits, reading from the left:

\( n \) 1’s, followed by one 0, followed by \( n \) 8’s, followed by one 9.

For example,
\[
A_2 = 110889
\]
and
\[
A_6 = 11111088888889.
\]

Prove that all of the numbers \( A_n \) are perfect squares. Your solution must include a complete proof.

Solution. Let \( A_n \) be the number whose digits, reading from left to right, are \( n \) 1’s, followed by one 0, followed by \( n \) 8’s and ending with one 9:
\[
A_n = \underbrace{11\ldots11}_{n \text{ 1's}} \underbrace{088\ldots88}_{n \text{ 8's}} 9.
\]

Then
\[
A_n = 9 + 8 \sum_{k=1}^{n} 10^k + \sum_{k=n+2}^{2n+1} 10^k
\]
\[
= 9 + 8 \left( \frac{10^{n+1} - 10}{10 - 1} \right) + \left( \frac{10^{2n+2} - 10^{n+2}}{10 - 1} \right)
\]
\[
= \frac{10^{2n+2} - 2 \cdot 10^{n+1} + 1}{9}
\]
\[
= \left( \frac{10^{n+1} - 1}{3} \right)^2.
\]