Problem 14. Let \( a_1 = 1 \), and for positive integer \( n > 1 \) define \( a_n \) recursively by

\[
\frac{a_n}{n + 1} = \frac{1}{n - 1} \sum_{k=1}^{n-1} a_k.
\]

Find the exact value of \( a_{2012} \).

Solution. Let \( S_n = \sum_{k=1}^{n} a_k \), so \( a_n = S_n - S_{n-1} \). The recursion in the problem can be rewritten as

\[
S_1 = 1 \quad \text{and} \quad \frac{S_n - S_{n-1}}{n + 1} = \frac{1}{n - 1} S_{n-1}, \quad n \geq 2.
\]

Thus, for \( n \geq 2 \) we have

\[
S_n = \frac{2n}{n - 1} S_{n-1}.
\]

It follows that

\[
S_n = \frac{(2n)(2(n-1))(2(n-2)) \cdots (2 \cdot 2)}{(n-1)(n-2) \cdots (1)} S_1 = \frac{2^{n-1} n!}{(n-1)!} = 2^{n-1} n.
\]

Therefore for \( n \geq 2 \)

\[
a_n = 2^{n-1} n - 2^{n-2} (n - 1) = 2^{n-2} (2n - (n - 1)) = 2^{n-2} (n + 1).
\]

Hence \( a_{2012} = 2^{2010} (2013) \).