A Gloss on the Book
“Laws of Chaos”
by Farjoun and Machover

Kenneth R. Driessel
June 19, 2014

Table of Contents

• Introduction

• Probability and random variables

• Rate of profit and rate of labour cost as random variables

• Labour-content as a measure of commodities

• The price and wage as random variables

• Dynamics

• References
Introduction

The dictionary on my computer has the following definition of the word “gloss”: “an explanation, interpretation, or paraphrase”. These notes provide a gloss on the book Laws of Chaos: A Probabilistic Approach to Political Economy by Emmanuel Farjoun and Moshe Machover (1983) (We use “FM” to briefly denote this book).

We shall discuss the mathematical models in economics presented in FM. We shall regard economics as a subject within applied mathematics. McKenna and Rees (1992) write:

It is now established beyond doubt that economics is a ‘mathematical’ subject. Though it is perfectly possible to learn economics in a non-mathematical way, one is always then reading a translation rather than the original and that is usually second best. The reason economics is mathematical is not that economists want to seem ‘scientific’ . . ., but because of its innate nature. It is concerned with quite long chains of reasoning about the interactions among numerical variables in complex systems, and mathematics is the most powerful and effective language we have for doing this.

In fact economics can be thought of as a collection of models. The process of doing, as opposed to talking about, economics, is entirely one of formulating analyzing and interpreting models.
Probability and Random Variables

**Definition:** A measurable space, \((S, \Sigma)\), consists of a set \(S\) ("the sample space") and a "sigma algebra", \(\Sigma\), of subsets of \(S\) (that is, \(\Sigma\) is closed under complements, intersection and countable unions).

**Definition:** A measure space \((S, \Sigma, \mu)\) consists of a measurable space \((S, \Sigma)\) and a "measure", \(\mu : \Sigma \rightarrow [0, +\infty)\), in other words, a function \(\mu\) that satisfies

\[
\mu(\bigcup_i A_i) = \sum_{i=1}^{\infty} \mu(A_i)
\]

provided the sets \(A_i\) are pairwise disjoint: \(i \neq j \Rightarrow A_i \cap A_j = \emptyset\).

**Definition:** A probability space is a measure space \((S, \Sigma, \mu)\) with \(\mu(S) = 1\). In this case we write \(P\) or \(p\) in place of \(\mu\).

In the examples of probability spaces in FM the sample space \(S\) is finite and the sigma algebra \(\Sigma\) consists of all the subsets of \(S\): \(\Sigma = \text{Power}(S)\). Then we have an "atomic" Boolean algebra and we can define the probability by specifying \(p(s)\) for elements \(s\) of \(S\).

**Definition:** A function \(X : S \rightarrow \mathbb{R}\) is a random variable on a probability space \((S, \Sigma, P)\) if, for all open intervals \((a, b) \subseteq \mathbb{R}\), the set \(X^{-1}(a, b) := \{s \in S : X(s) \in (a, b)\}\) is in \(\Sigma\). Then the function \(F_X := \mathbb{R} \rightarrow \mathbb{R} : r \mapsto \mu(X^{-1}(-\infty, r))\) is the cumulative distribution function (c.d.f.) of \(X\) and the derivative \(f_X := (d/dr)F_X(r)\) is the probability density function (p.d.f.) of \(X\). The expected value of \(X\), denoted \(EX\), is defined by

\[
EX := \int_{-\infty}^{\infty} r f_X(r)dr.
\]
The variance of $X$, denoted $VX$, is defined by

$$VX := \int_{-\infty}^{\infty} r^2 f_X(r)dr - (EX)^2.$$ 

**Rate of profit and the rate of labour cost as random variables**

(See FM pp.62-68.)

The firm space consists of the finite set, Firms, of firms together with probability function

$$p := \text{Firms} \to [0, 1] : i \mapsto K(i)/\text{Sum}(K)$$

where $K(i)$ is the amount of capital of firm $i$ and

$$\text{Sum}(K) := \sum\{K(i) : i \in \text{Firms}\}$$

is the total capital of the firms. We shall assume that each firm has a positive amount $K(i) > 0$ of capital.

Question: Is this positivity condition needed? It might be convenient to have some firms “die” which could be represented by $K(i) = 0$.

Using this probability space we can define the rate of profit as the following random variable. Fix a time $t$. Define

$$R := \text{Firms} \to \mathbb{R} : i \mapsto R(i)$$

where $R(i)$ is the rate of profit firm $i$ per unit of capital at time $t$. More precisely, let $h$ be a short time period (e.g. a month); then $hR(i)K(i)$ is approximately the profit earned by firm $i$ in interval $(t, t + h)$. (Note that
\( R(i) \) has units \( 1/(time \times money) \) and \( hR(i)K(i) \) is dimensionless.) We have associated functions \( F_R \), the cumulative distribution function of \( R \), and \( f_R \), the probability density function of \( R \).

Using the firm space we can define the **rate of labour cost** as the following random variable:

\[
Z := \text{Firms} \rightarrow \mathbb{R} : i \mapsto Z(i)
\]

where \( Z(i) \) is the rate of labour cost of firm \( i \) per unit of capital at time \( t \).

More precisely, let \( h \) be a short time period (e.g. a month); then \( hZ(i)K(i) \) is approximately the labour cost spent by firm \( i \) in interval \( (t, t+h) \). (Note that \( Z(i) \) has units \( 1/(time \times money) \) and \( hZ(i)K(i) \) is dimensionless.)

TODO: Include a paragraph on \( e_0 := ER/EZ \).

**Labour-content as a measure of commodities**

Pick a (short) time period \( h \) (e.g., one day). Let \( C \) be a commodity-type (e.g., sugar). Let \( C-Sales := \{1, \ldots, n\} \) be the space of transactions that occur during the time period \( h \) in which the commodity \( C \) was sold-bought. We pick a unit, \( C \)-unit, for measuring quantities of \( C \) (e.g., kilograms for sugar).

Let \( Q(i) \) denote the quantity sold at transition \( i \). Let be \( p := C-Sales \rightarrow \mathbb{R} \) be the function defined by

\[
p(i) := \frac{Q(i)}{\text{Sum}(Q)}
\]

where \( \text{Sum}(Q) := \sum\{Q(i) : i \in C-Sales\} \) is the total sales. We have defined in this way the **C-Sales space**.

Define the function \( S := C-Sales \rightarrow \mathbb{R} : i \mapsto S(i) \) where \( S(i) \) is the sale price per unit in the \( i \)th transaction. Note \( S(i)Q(i) \) is the total price
paid-sold in the $i$th truncation. We have thus defined a random variable $S$ on the space of C-sales. Note the $S(i)$ has units $\text{money}/C$ – unit. For example, we have the sugar price random variable defined on the space of Sugar-Sales; it has units $\text{money}/\text{kilograms}$. As another example, we have the petrol price random variable defined on the space of Petrol-Sales; it has units $\text{money}/\text{liters}$.

We need to face the “units problem”: How do we compare prices for different commodities? FM notes that we can use the “input-output” theory of Liontief to provide a solution of this problem once we select a “universal commodity” as a “yardstick”. In the next section we select (following FM) labour-content as the universal quantity.

**Price and wage as random variables**

We divide the set of all transactions in two discount sets: the set of labour transactions and the set of “general commodity transactions”. In other words, the “general commodity transactions” are the ones that are different than labour transactions.

Let $T$ be a “reference period” (e.g., one month). The labour-space consists of the set $L$ of all labour transitions \( \{L(i), i = 1, \ldots, n\} \) in worker-hours during $T$. Define the function

\[
p := L \to \mathbb{R} : i \mapsto L(i)/\text{Sum}\, L.
\]

In other words, the transactions $L(i)$ are “equal weighted”.

We define the “wage” random variable $W$ by

TODO: Include a definition of the ave unit wage.
The **market space** consists of the set \( M \) of all general-commodity transitions \( M = \{ M(i) := \Lambda(i), i = 1, \ldots, n \} \) during \( T \). We measure \( M(i) = \Lambda(i) \) in terms of labour content.

Define the function \( p := M \to \mathbb{R} : i \mapsto \Lambda(i)/\text{Sum}\Lambda \).

We define the “specific price” random variable \( \Psi \) by

\[
\Psi := M \to \mathbb{R} : i \mapsto \Pi(i)/
\]

where auw is the “average unit wage”.

References