Stability analysis of non-equilibrium behavior in nonlinear dynamical systems

Umesh Vaidya
Electrical and Computer Engineering Department
Iowa State University

Recently there has been increased interest in the study of non-equilibrium behavior in natural and applied sciences. Most of the biological systems are known to operate away from equilibrium. Phenomena of self assembly in nano systems, self organization and synchronization in biological systems, mixing in fluid flows are some of the examples where the steady state is characterized by non-equilibrium dynamics. Analysis and control of the non-equilibrium behavior is a challenging problem but with number of exciting applications and promising future technological impact. In this talk we present theoretical and computational framework for the verification of global stability of a steady state non-equilibrium behavior for a nonlinear dynamical system or a continuous mapping. Using methods based on stochastic dynamics, a *Lyapunov measure* is proposed for this purpose. Lyapunov measure is shown to be stochastic counterpart of stability (transience) just as invariant measure is a counterpart of attractor (recurrence). Lyapunov measure generalizes the concept of Lyapunov function that is typically used for verifying stability of an equilibrium in nonlinear systems. In particular we show that the Lyapunov measure is dual to Lyapunov function. Methods for computing these Lyapunov measures are presented based upon set-oriented numerical approaches. These methods arise as a series formula and a linear program that allow one to carry over the intuition from the linear case with stable equilibrium to general nonlinear systems with globally stable attractor sets.