Equitable colorings of sparse graphs

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In several applications of coloring as a partition problem there is an additional requirement that color classes be not so large or be of approximately the same size. Examples are mutual exclusion scheduling problem, scheduling in communication systems, construction timetables. A model imposing such a requirement is *equitable coloring*—a proper coloring such that color classes differ in size by at most one. Pemmaraju and Janson and Ruciński used equitable colorings to derive deviation bounds for sums of dependent random variables that exhibit limited dependence. Rödl and Ruciński used equitable colorings to give a new proof of the Blow-up Lemma.

In contrast to ordinary coloring, a graph may have an equitable k-coloring (i.e., an equitable coloring with k colors) but have no equitable $(k+1)$-coloring. Thus, it is natural to look for the minimum number, $\mathrm{eq}(G)$, such that for every $k \geq \mathrm{eq}(G)$, G has an equitable k-coloring. Finding $\mathrm{eq}(G)$ even for planar graphs G is an NP-hard problem. This motivates a series of extremal problems on equitable colorings. In this talk we discuss some recent progress on equitable coloring of sparse graphs.

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